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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

CHAOS SYNCHRONİZATİON İN SYSTEMS WİTH FEEDBACKS

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GENERAL DESCRIPTION OF THE WORK

Actuality of the work and work done so far. A lot of non-linear dynamical systems upon some diapason of systems' parameters exhibits chaotic behaviour. Such systems exist almost in all of the science branches such as physics, chemistry, biology, economy, sosiology, etc. Chaotic dynamics is possible in both continuous and discrete systems, even in one-dimensional discrete models. This fact is of immense fundamental importance, as it is clear that low dimensional systems behave as a statistical systems. The discovery of chaotic dynamics has meant huge blow to Laplasian determinism after quantum mechanics.

Chaos may exist in nonlinear systems. The main reason for such behaviour are the unstability of the systems for some parameter values of the systems and our inability to determine the inital conditions with infinite precision.

From the matemathical point of view chaos is generated when the Lyapunov exponent is positive; this exponent shows the rate of divergence of the close trajectories with time. In other words, chaotic systems are very sensitive to the small change of the initial conditions for the systems. Sometimes it is called the butterfly effect. As an example, the small movement of the butterfly's wing in the South America could generate air movement which can be trnsformed into the tornado in the North America. An attractor of the chaotic dynamics (where with the time all the system's trajectories approaches and stays there) is of fractal nature. Trajectories also should be bounded and cannot go to the infinity.

Currently it is proved that chaotic dynamics exists in many nonlinear systems of interdisciplinary nature. In other words, such dynamics is one of important features of the system' s evolution and is widespread in both natural and man-made systems. In general there is no a single recipe for the determination of usefullness or harmness of such dynamics, and it depends on the persued aim. Thus in some cases the existense of chaotic dynamics can accelerate the chemical reactions, heat and matter exchange. In some cases the presence of chaotic dynamics can be undesirable. For example in construction works chaotic vibrations can facilitate the mechanical damage of the construction. Also, from the phycological point of view the presence of chaos could be the reason the the tiredness and excruciating uneasiness .

One of the important features of the chaotic dynamics is its extreme sensitivity to the accuracy of the initial conditions. From the one side such a sensitivity makes the long-time prediction impossible; on the other side this sensitivity makes it easy to achieve some goals with less energy. Lately there has been a huge progress in controlling chaotic dynamics. Under this term it is understood the conversion of chaotic osscilations into the peroidic, quasi-periodic states or the stationary states of the system and vice-versa. Such control can be realised with the help of timely intervention in the system's parameter values or in the dynamical variables itself.

Synchronization of chaotic osscilations between two and more systems is a another way of controlling chaos. In the end during such synchronization the trajectories of the systems osscilate in unison.

Control of chaotic dynamics can be used in secure communications, in optimization on the nonlinear systems activities, etc.

In secure communications synchronization between the transmitter and receiver systems is very important for the information message decoding at the receiver end.

Figure 1 shows the principles of information exchange based on chaos synchronization. At the transmitter system message is masked with chaotic dynamics and these signals sum is sent to the receiver system. This summary signal before reaching the receiver is divided into two parts. At the receiver end due to the chaos synchronization between the transmitter and receiver chaos is regenerated. By deducting the receiver output from the summary signal the transmitted signal is recovered. Thus synchronization between the transmitter and receiver systems allows for the message recovery for chaos-based communication systems.

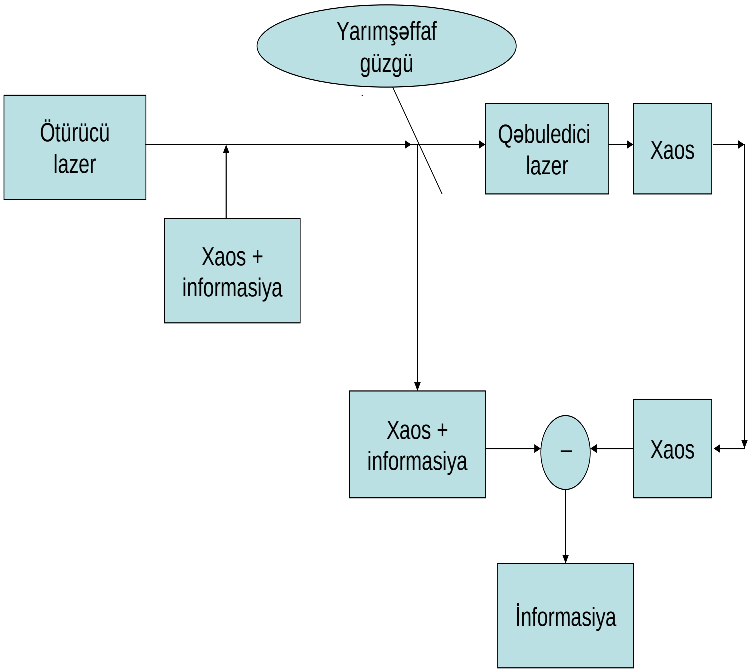

Figure 1. The principal scheme of chaos-based communication system

In the traditional cryptographic systems for secure communication the software programs are used. But as nowdays the computers speed (CPU-komputer' processing units) is increased yearly, it poses certain risks for message transmission and information processing. The use of laser chaos for the secure communication offers additional layer of reliability –hardware cryptography. In this case there is additional requirement to synchronize transmitter and receiver lasers with 200-300 parameters. In real time communication such a requirement presents additional security, as itis nearly impossible for the unauthorized intruder to decode the transmitted message. Laser chaos based secure communication is considered a cost effective, high speed and high security alternative communication channel for information processing.

Synchronization of laser systems is important for secure communication application perspectives. Additionally such synchronization can afford for compact, high intensity laser sources. Moreover synchronization between laser systems, controlling the synchronization error is important from the point of view of designing a dynamically transforming logic elements, which could bring eventually to the designation of compact and high speed computer systems.

Synchronization of two or more systems and transition between synchronization and desynchronization might have implcations for medical applications. According to some scientific reports, one of the reasons for the epilepsy disease could be synchronization of neurons in human brain.
Synchronization-desynchronization transition could be important from the viewpoint of preventing the spread of infectious diceases. From the fundamental point of view synchronization between the complex system elements can reduce the uncertanity along some directions in computer simulation of such systems.

As mentioned above chaotic dynamics exists in both continous and discrete systems. Ordinary differential equations exhibit chaos if there are at least three dynamical variables. As for the discrete systems chaos may exist even one dimensional systems. It is noted that for secure communication schemes such systems cannot guarantee high level security.

For this reason lately from the secure communication application viewpoint functional differential systems are receiving more attention, as such systems can provide high dimensional chaos (hyperchaos). Delay differential equations also belong to functional differential systems. In such systems the current state is determined by the system’s previous state. Such kind of systems are more realistic as they take into account the boundedness of the information exchange speed, swithching speed, etc.

Delay differential equations arise in modelling of systems with feedback(s). As already mentioned delay differential equations generate hyperchaos, they are systems with two or more positive Lyapunov exponents. The number of positive Lyapunov exponents can increase with feedback time. From the mathematical point of view delay differential equations are of infinite dimensional, as for such systems the inital conditions are given not at some time, but in the some time interval. From this point of view such systems can be used in modelling of processes described by the partial differential equations.

Thus it should be noted that although synchronization between the functional systems with a single feedback is studied relatively in detail, synchronization between systems with multi feedbacks is hardly widely studied. Especially it is true for systems with modulated feedbacks and modulated coupling delays.

Study of synchronization between the Josephson junctions is also relatively new direction in the field. Such synchronization of the Josephson junctions is vital for achieving practically adequate sources of power in the terahertz region. Additionally such sources could be compact, mobile, transferable, cost effective with small sizes.

In the dissertation work synchronization between the chaotic systems with constant and modulated feedback(s) and connected with coupling delay (s) is researched.

The aim of this dissertation is to research synchronization regimes between the systems with time delays with one or several constant and/or modulated feedbacks. Also in this work synchronization between the systems coupled both with constant and modulated time delay(s) is studied.

To achieve this aim in the dissertation the following goals are achieved.

- To elucidate the necessary and sufficient conditions for synchronization regimes between a single time delay systems
- To determine the effect of parameter mismatches between the synchronized systems on the synchronization quality.

- To find the necessary and sufficient conditions for synchronization regimes between the systems with several feedbacks.
- To elucidate conditions when parameter mismatches between the synchronized systems can play constructive role to achieve synchronization.
- To show that synchronization is possible between the systems with modulated feedbacks and coupling time delays, including chaotic modulations
- To determine conditions for the generalized synchronization between systems with parameter mismatches.
- To establish the possibility of synchronization between Josephson junctions coupled in series.

In this dissertation the models under investigation are: Ikeda model, Mackey-Glass model and Josephson junctions. These models are very popular in non-linear chaotic dynamics.

Historically for the first time Ikeda model was used to describe the dynamics of the light-electromagnetic waves in the optical bi-stable resonator. Later it was established that such a model can be adequate in studying the dynamics of B-class laser systems, which include some semiconductor lasers, solid state lasers, low pressure CO_2 lasers. It is well-known that according to the Maxwell –Bloch equations there are four types of laser systems: A,B,C,D. This division is dependent on the relationships between the relaxation times for polarization, electrical field and inverse populations.

Mackey-Glass systems were first used to describe the dynamics of leucocytes, erythrocytes in human blood. It is well-known that for healthy persons the number of leucocytes is fluctuated around average value of leucocytes with small amplitudes. Quite contrary in the hematological patients these fluctuations occur with larger amplitude and dynamics is repeated after 20-30 days. Studying Mackey-Glass model dynamics could shed a light on how to correct the pathological abnormalities with due-time intervention. It is worth mentioning that there exists the electrical scheme for

Mackey-Glass systems which makes the study of dynamics experimentally easy. This fact also underlines that Mackey-Glass systems are of interdisciplinary nature.

Josephson junctions are also one of popular dynamical systems in nonlinear physics. For such systems for some parameter values there can be a chaotic behavior. Investigation of chaos in such systems is important from the viewpoint of both fundamental and application perspectives. Controlling chaos in Josephson contacts helps to use these systems as a detector, voltage standard, in secure communications, the source of terahertz radiation, etc.

Investigative methods used in this work: models under investigation are studied with contemporary mathematical methods, such as Lyapunov-Krasovskii and Lyapunov-Razumikhin functional used to analyze the stability of the delay-differential equations. The analytical results were supported by the extensive numerical simulations and an agreement between the two was excellent. Numerical simulations were performed with the help of Matlab 7 software DDE23 and DDESD.

Main scientific results to be defended are the followings:

- ✓ There are different types of stable synchronization regimes between the systems with a single feedback.
- ✓ In certain cases parameter mismatches between the systems to be synchronized are necessary to achieve synchronization.
- ✓ The number of synchronization regimes between the multi-feedback systems exceeds the number of synchronization regimes between the single feedback systems.
- ✓ It is possible to acheive synchronization between the systems with modulated feedback times and coupling times.
- ✓ Under parameter mismatches generalised synchronization between time delay systems is possible.
- ✓ Synchronization is possible between the Josephson junctions coupled with time delays unidirectionally and in series.

Scientifically **new results** of the dissertation can be summarized as following:.

- Stability conditions between chaotic systems with a single feedback are found.
- It is shown that the parameter mismatches between the systems to be synchronized can play constructive role.
- Possible synchronization regimes between the systems with several feedbacks are elucidated. It is established that the number of synchronization regimes for systems with multifeedback exceed the number of those available for systems with a single feedback.
- It is found that under conditions of chaotic and/or harmonic modulations of the feedback time delays and coupling time delays synchronization is still possible.
- Under the conditions of parameter mismatches generalised synchronisation between different systems is possible.
- It is establshed that synchronization between unidirectionally coupled Josephson contacts in series configuration can be achieved. Synchroniation quality between the Josephson junctions depends significantly on the coupling time between the junctions.

Theoretical and practical significance of the dissertation work. The research in this dissertation imply that simple, one dimensional systems under certain conditions can exhibit chaotic, fluctuation type dynamics. This result is of certain importance in the generation of random numbers. In complex systems the presence of synchronization regimes can be helpful in computer modeling of such systems. Synchronization of systems can be used in morphing into each other logic elements, which could help to increase the computer processing speeds.

The results of this work can be used for chaos based secure communication systems and information processing. Currently security in such systems is provided by means of factorization of huge numbers. With ever increasing computing speeds this approach could fail very soon. Ideal quantum systems based security is still to far from the practical application. Laser chaos based security could provide additional security belt for current schemes of security.

Synchronization between Josephson junctions is important from the viewpoint of achieving of an adequate power for applications in numerous areas of scientific and technological world in the Terahertz diapason.

Thesis approbation: The results of this disseration are presented at the VI Republican scientific conference on physics (Baku, 13-15 December, 2012) and at the seminars of the Institute of Physics of Azerbaijan National Academy of Sciences.

Institution where this dissertation work was performed. The work was performed at "Chaos in Dynamical Systems" laboratory of the "İnnovation" Sector with the Institute of Physics of Azerbaijan National Academy of Sciences

Published papers: There are 11 published papers for this dissertation. The list of these papers is given at the end of this ABSTRACT.

The volume of dissertation structural units in characters and total volume of dissertation in characters . Title page-400 characters; contents- 2435 characters; introduction-24360 charaters; main part of dissertation-168 045 characters;results section- 4652 characters; references list-21583 characters; abbreviation section-580 characters. Total volume of this dissertation (with the exception of pictures, tables, graphs, additional pages, reference list)-**200,000** characters. Overall dissertation volume-221 970 characters.

Overall this dissertation contains Introduction, VI Chapters, main results Section and the List of References. All in all this work contains 173 pages, including 45 Figures and List of 152 references.

Applicant's own contribution to the obtained results: With the exception of two papers, results of this dissertation are published with several other authors. At the same time it should be noted that main scientific results are obtained with the direct participation and leadership of the dissertant. The author of this dissertation took leading part in establishing of the scientific problems, in the application of the mathematical methods to solve the problems, obtaining of the results, their discussions and in the process of manuscript preparation for publication.

BRIEF SUMMARY OF THE DISSERTATION

In the Introductory part of the dissertation the actuality of dissertation subject is emphasized, the purpose of the work is determined, obtained new results and their practicality are shown.In this part of dissertation the results of the dissertation and chapter by chapter brief essence of the dissertation is also presented.

Chapter I of the dissertation deals with the review of the available scientific literature on chaos control. Main attention is given to the case of chaos synchronization between functional systems-systems with feedbacks and coupled with some time delay. In the center of this chapter is dynamics of the Ikeda and Mackey-Glass models and of the Josephson junctions. At the end of Chapter I the purpose of the dissertation theme is presented.

In Chapter II of the dissertation existence and stability conditions of the synchronization regimes between the unidirectionally and nonlinearly coupled Ikeda systems are analyzed. Main attention is given to the anticipation synchronization. In anticipating synchronization the driven system synchronize to the future state of the drive system. In other words the driven system anticipates the dynamics of the driver system. It should be noted that this case is asymptotic and reason and conclusion sequence (casulity principle) is not violated.

In Chapter II the existence and stability conditions of the anticipating synchronization is found by use of Lyapunov-Krasovskii functional method for functional time-delay equations.

Consider the synchronization between the driver (master) system $x(t)$ and driven system (slave) $y(t)$:

$$\frac{dx}{dt} = -\alpha_1 x + m_1 \sin x_{\tau 1} \tag{1}$$

$$\frac{dy}{dt} = -\alpha_2 y + m_2 \sin y_{\tau 2} + m_3 \sin x_{\tau 3} \tag{2}$$

Here $x(\tau) \equiv x(t-\tau)$; α - relaxsation rates; m_1 and m_2 - are the intensities of the injected to the optical resonators lasers; τ_1 və τ_2 - are the feedback times for the driver and driven systems; m_3 - is the coupling intensity between the synchronized systems. This coupling is not instantatious and occurs with the delay τ_3 . It is established that under conditions $\alpha_1 = \alpha_2$, $\tau_1 = \tau_2$, $m_1 = m_2 + m_3$ $x = y_{\tau_1 - \tau_3}$ (or $y = x_{\tau 3 - \tau 1}$) is the synchronization regime. Stability condition of this regime is can be written as: $\alpha > |m_2|$. $\tau_1 > \tau_3$ is the case of anticipating synchronization: $y(t) = x(t - \tau_3 + \tau_1)$ $\tau_1 < \tau_3$ - is the case of lag synchronization and $\tau_1 = \tau_3$ is the case of identical (complete) synchronization. To characterize the quality of synchronization the correlation coefficent can be used:

$$C(\Delta t) = \frac{\langle (x(t) - \langle x \rangle)(y(t+\Delta t) - \langle y \rangle) \rangle}{(\langle (x(t) - \langle x \rangle)^2 \rangle \langle (y(t+\Delta t) - \langle y \rangle)^2 \rangle)^{1/2}} \tag{3}$$

$\langle . \rangle$ -indicates time averaging; Δt is the shift between the synchronized systems, e.g. in an anticipating synchronization $\Delta t = \tau_1 - \tau_3$.

C=± 1 evidences the case of high-level synchronization. In Figure 2 the dynamics of the driver x and driven y sistems is shown for parameter values $\alpha = 2$, $m_1 = 10$, $\tau_1 = 5{,}5$, $\tau_3 = 0{,}5$,

$m_2 = 0{,}2$, $m_3 = 9{,}8$. It is seen that the driven system's trajectory (broken line) anticipates the trajectory of the driver system by system $\Delta t = \tau_1 - \tau_3$ time shift.

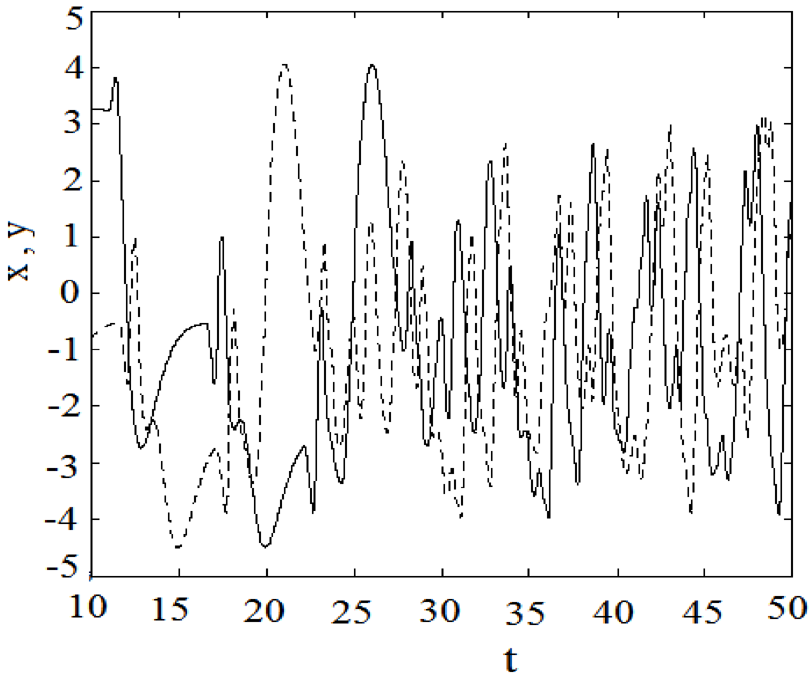

Figure 2. The driven system's trajectory (broken line) anticipates the trajectory of the driver system by system $\Delta t = \tau_1 - \tau_3 = 5$ time shift.

Chapter II ends with the results of computer simulations on the effect of parameter mismatches on the synchronization quality. General trend is that such effect may bring to the slight deterioration of the synchronization quality.

Figure 3 depicts the effect of the feedback time mismatches on the synchronization quality in terms of the correlation function C. This case corresponds to the anticipating synchronization between the driver and driven systems with parameters as in for Figure 2.

It should be emphasized that case considered in Chapter II is just one aspect of the parameter mismatches influence on the synchronization quality. As shown in the following chapters in some specific cases parameter mismatches between the synchronized systems even is essential to achieve synchronization. In other words, there are cases when parameter mismatches can play not destructive, but constructive role in the synchronization phenomenon.

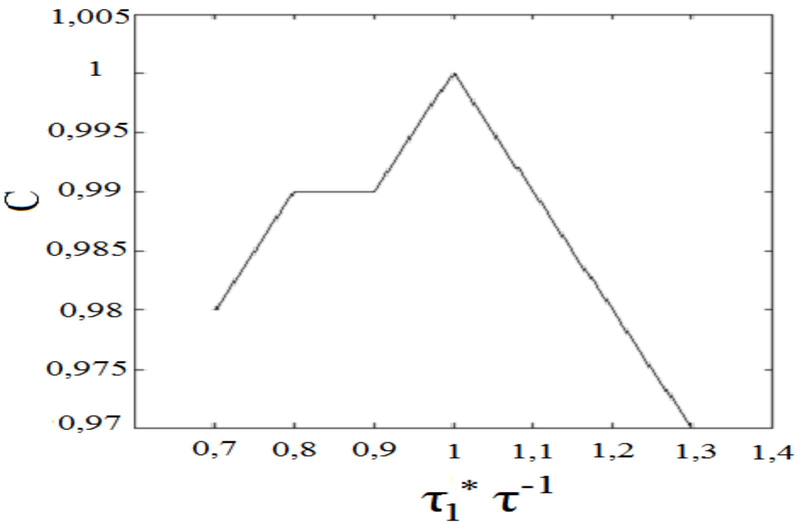

Figure 3. Dependence of the correlation function C on the $\tau_1^* \tau^{-1}$, τ_1^* and τ are the feedbck times for the driven and driver Ikeda systems correspondingly.

In Chapter III the effect of parameter mismatches on the synchronization quality is studied in detail. As an example synchronized systems are coupled unidirectionally and bi-directionally with some delay.

Synchronized systems are chosen of both general nature and in the form of famous Ikeda and Mackey-Glass nonlinearities. It should be underlined that parameter mismatches between the synchronized systems act on the synchronization quality multifacially. In some cases this action is negative which gives rise to the deterioration of the synchronization quality. In certain cases the parameter mismatches change the type of synchronization from complete to lag synchronization. With larger values of parameter mismatches synchronization between system may disappear.

In Chapter III completely new scenario of the effect of parameter mismatches on the synchronization quality is demonstrated: Parameter mismatches between the synchronized systems are the only way to achieve synchronization. Moreover only the lag synchronization between the synchronized systems is permitted; other types of synchronizations such as complete, anticipating, etc. are not possible. Theoretical results are fully supported by numerical simulations. The results of this Chapter demonstrate that choosing certain types of parameter mismatches and coupling between the synchronized systems one can selectively create certain rejime of synchronization with other types of synchronization being forbidden. Such a selective synchronization may have a wide range of application perspectives. It is possible that such a selective synchronization can play certain role in information processing and exchange in live natural systems.

Chapter III also dealt with the synchronization of systems with variable feedback time delay systems. At the same time there was no modulation of the coupling time between the systems. In general form this problem was the subject of the detailed investigation in Chapter IV.

Thus in Chaper III it is established that by choosing the coupling type and certain parameter mismatches one can achieve a selective synchronization. For such a case in Chapter III both the existence and stability conditions are found.

Considering the synchronization forms between the driver

$$\frac{dx}{dt} = -\alpha_1 x + m_1 f(x_{\tau_1}) \tag{4}$$

and driven systems

$$\frac{dy}{dt} = -\alpha_2 y + m_2 f(y_{\tau_1}) + m_3 x_{\tau_2} \qquad (5)$$

with the linear coupling
it was established that under conditions $m_1 = m_2$ və $\alpha_2 - \alpha_1 = m_3$

$$x_{\tau_2} = y \qquad (6)$$

Lag synchronization is possible and this synchronization form is stable under condition:

$$\alpha_2 > \left| m_1 f^{'}(x_{\tau_1+\tau_2}) \right| \qquad (7)$$

Where $f^{'}$ is the derivative of f with respect to x.

It should be noted that for the Ikeda model $f(x_\tau) = \sin(x_\tau)$ and for the Makey-Glass model $f(x_\tau) = \frac{x_\tau}{1 + x_\tau^{\ b}}$; here b is some constant, usually $b = 10$.For these cases stability condition (7) accept the form for the Ikeda model $\alpha_2 > |m_1|$ and for the Makey-Glass model this condition is of the form $\alpha_2 > \left| m_1 \frac{(b-1)^2}{4b} \right|$.Thus independent of the relationship between the feedback times and coupling times only lag synchronization occurs.

Results of computer simulations fully support the analytical findings. Figure 4 depicts dynamics of driver and driven Ikeda models for parameter values $\alpha_1 = 1$, $\alpha_2 = 6$, $m_1 = m_2 = 4$, $m_3 = 5, \tau_1 = 3, \tau_2 = 10.$

It is noted that these parameter values satisfy both the existence and stability conditions for lag synchronization $y = x_{\tau_2}$ mentioned above.

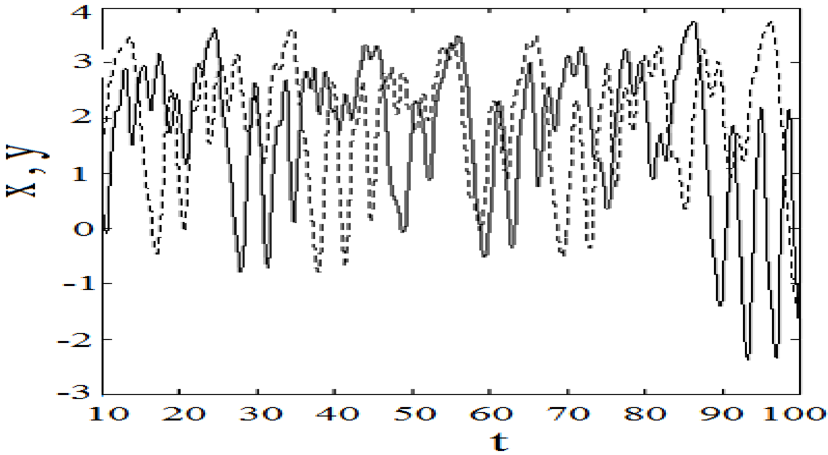

Figure 4. Under conditions of different relaxation rates dynamics of the driven system y (dotted line) lags behind the dynamics of the driver system x (solid line) by $\tau_2 = 10$ time steps.

It is observed that according to the theoretical results the driven system trajectory lags behind the trajectory of the driver system by 10 time units: $y(t) = x(t - \tau_2) = x(t - 10)$.

Figure 4 corresponds to the case $\tau_1 < \tau_2$. As computer simulations show only lag synchronization occurs independent of the relationship between the τ_1 and τ_2.

Finally in Chaper III it is shown that stable synchronization is possible under the condition of modulation of the feedback times. Using the Lypunov-Razumukhin functional approach it is found that lag synchronization $x_{\tau_2} = y$ is stable if

$$\alpha_2^{\,2}\left(1 - \frac{d\tau_1(t)}{dt}\right) > \left(m_1 f^{'}(x_{\tau_1(t)+\tau_2})\right)^2 . \qquad (8)$$

In Chaper IV synchronization between the systems with several feedbacks is investigated and existence, stability conditions for the possible synchronization rejimes are found. It should be noted that systems with several feedbacks are widespread in nature and science. In laser systems additional feedback mechanism can be used for the stabilisation of the laser intensity. In nature such feedbacks could be of some impotance for the adaptation purposes. Additionally it is stipulated that laser systems with several feedbacks could provide more complex chaos more suitable for the encryption of the messages at chaos based communication scheme. Moreover it is proved that systems with several feedbacks can provide more positive Lyapunov exponents with higher values. Based on these factors, Chaper IV is dedicated to synchronization between systems with feedbacks.

Importantly In Chaper IV it is found that for systems with feedbacks the number of the synchronization rejimes exceeds the number of those for systems with a single feedback.

In Chaper IV of the dissertation synchronization between the systems with feedbacks are performed for the Ikeda and Mackey-Glass systems. Both the linear and non-linear types of coupling between the synchronized systems are investigated. Here only the results of the investigation of synchronization types between Mackey-Glass systems with two feedbacks coupled unidirectionally are presented.

Suppose that the driver system with two feedbacks

$$\frac{dx}{dt} = -\alpha\ x + k_1 \frac{x_{\tau_1}}{1 + x_{\tau_1}^b} + k_2 \frac{x_{\tau_2}}{1 + x_{\tau_2}{}^b} \tag{9}$$

drive the driven Mackey-Glass system

$$\frac{dy}{dt} = -\alpha\ y + k_3 \frac{y_{\tau_1}}{1 + y_{\tau_1}^b} + k_4 \frac{y_{\tau_2}}{1 + y_{\tau_2}{}^b} + K \frac{x_{\tau_3}}{1 + x_{\tau_3}{}^b} \tag{10}$$

It is clear that synchronization rejime $y = x_{\tau_3 - \tau_1}$ between these systems is possible under conditions $k_1 = k_3 + K, k_2 = k_4$ .Stability condition of this synchronization form $y = x_{\tau_3 - \tau_1}$ can be written as: $\alpha > (k_1 + k_2)\frac{(b-1)^2}{4b}$. Existence conditions for the synchronization form $y = x_{\tau_3 - \tau_2}$ are $k_2 = k_4 + K$, $k_1 = k_3$. Stability condition is of the form: $\alpha > (k_2 + k_3)\frac{(b-1)^2}{4b}$. It is easy to check that the number of synchronization forms for this case double feedbacks is 6. At the same time that number is equal to 3 for a single feedback. Multitudeness of synchronization rejimes is desirable for the application point of view, such as in adaptive tasks.

For linearly coupled systems of the form $K(x - y)$ synchronization type $x = y$ is possible under existence conditions $k_1 = k_3, k_2 = k_4$. Synchronization rejime is stable under condition $\alpha + K > (k_1 + k_2)\frac{(b-1)^2}{4b}$. Analytical results are fully supported by numerical simulations. Figure 5 depicts synchronization between double feedbacks Mackey-Glass systems with the linear coupling.

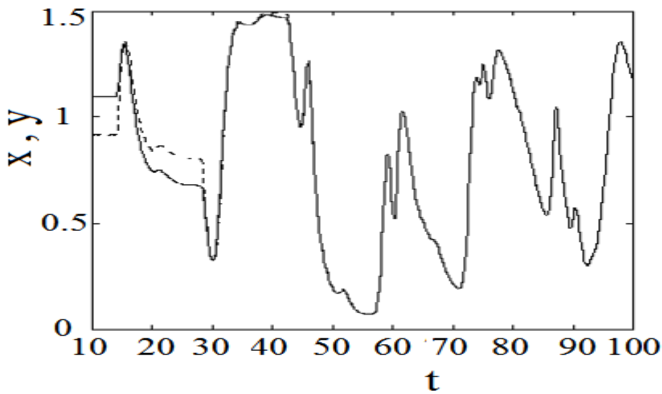

Figure 5. Dynamics of the driver (solid line) and driven (broken line) double feedbacks Mackey-Glass systems with linear coupling.

Correlation function between the systems $C=1$. The following parameter values are used for numerical modelling $\alpha=1, b=10, \tau_1=14, \tau_2=20, k_1=k_3=2, k_2=k_4=0.2, K=5$.

It should be noted that $K=5$ satisfies necessary stability condition. Under this condition the synchronization rejime is robust to the effect of noise and parameter mismatches. There is also minimal value of the coupling intensity when synchronization rejime is stable $K\approx 3.46$. This value is found from the calculation of the negativity condition of the Lyapunov exponent from the dynamics of the synchronization error $x-y$. For the coupling intensity found from this condition the synchronization rejime is extremely sensitive to noise, fluctuation, parameter mismatches, etc.

Chapter IV also deals with synchronization rejimes between the systems under the modulation of the feedback times and coupling times.

In this Chapter inverse synchronization rejime ($x=-y$) between the systems coupled unidirectionally and bidirectionally with linear or nonlinear coupling is studied in details.

Figure 6 shows invers synchronization between Ikeda systems with modulated double feedbacks coupled bidirectionally and nonlinearly. Coupling time is also modulated.

$$\frac{dx}{dt}=-\alpha x-m_1\sin x(t-\tau_1)-m_2\sin x(t-\tau_2)+K_y\sin y(t-\tau_3) \quad (11)$$

$$\frac{dy}{dt}=-\alpha y-m_3\sin y(t-\tau_1)-m_4\sin y(t-\tau_2)+K_x\sin x(t-\tau_3) \quad (12)$$

Where

$\tau_{1,2}=\tau_{01,02}+x_1(t)\tau_{a1,a2}\sin(\varpi_{1,2}t)$ are the modulated feedbacks' times;

$\tau_3=\tau_{03}+x_1(t)\tau_{a3}\sin(\varpi_3 t)$ are the coupling time modulations;

$\tau_{01,02,03}$ are the constants ;

$\tau_{a1,a2,a3}$ are the modulation amplitudes; $\varpi_{1,2,3}$ are the modulation frequencies; $x_1(t)$ is the solution of the İkeda system with constant double feedbacks' times; $m_{1,2}$ və $m_{3,4}$ are the feedback strengths for x və y systems correspondingly; $K_{x,y}$ are the coupling intensities between the systems x və y.

It is clear that modulated feedback and coupling times include both chaotic and harmonic components.

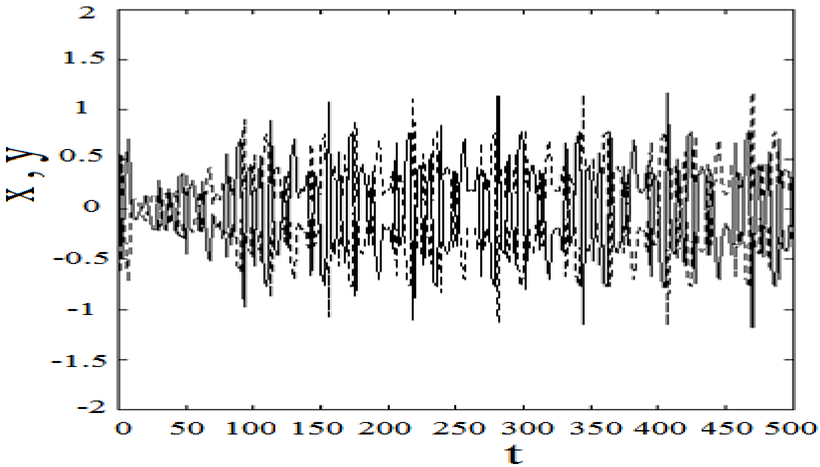

Figure 6. Inverse synchronization between the double feedbacks Ikeda systems coupled bidirectionally. Both feedback and coupling times are subject to modulation, including the chaotic component.

Numerical modelling is performed using the following parameter values:

$\tau_1(t) = 3 + 2x_1(t)\sin(0.15t), \tau_2(t) = 5 + 2x_1(t)\sin(0.15t)$ and

$\tau_3(t) = 7 + 2x_1(t)\sin(0.15t)$;

$\alpha = 3,\ m_1 = m_3 = 3.1,\ m_2 = m_4 = 2.5,\ K_x = K_y = 0.03$.

Chapter V of this dissertation is dedicated to the generalized synchronization between the Mackey-Glass and Ikeda models under conditions of parameter mismatches.In generalized synchronization there is a functional dependence between the systems to be synchronized. Such dependence is attractive from the point of view of secure communications, as for the message decoding one has to calculate the functional dependence between the synchronized sender and receiver systems. Usually such functional dependence is very complex and intricate, thus making it impossible for the unauthorized third party to find this function in order to decode the message sent from the transmitter system.

First in Chapter V by using the auxuilary system approach the conditions of the generalized synchronization between the Mackey-Glass and Ikeda systems are found using the Lyapunov-Krasovskii functional approach. Further, in *Chapter V* a general theory of the generalized synchronization between Ikeda and Mackey-Glass type systems is developed.

Chapter VI deals with the Josephson junctions and synchronization between the junctions. First brief information is given about the Josephson junctions and possible synchronization between them, and practical implications of such a synchronization. It is emphasized that Josephson junctions are important class of non-linear systems and these systems are capable of exhibiting chaotic behaviour upon some parameter values of the system. It is also noted that control of chaos including chaos synchronization in such systems are of certain practical importance. Controlling chaos in junction dynamics is important from the point of use of these junctions as a detector, voltage standards, etc. In such cases chaoticity id not desirable! Chaotic Josephson junctions can be used for secure communications for not long distances, for the determination of the distance from the object of interest with high precision. From the other side, it is well-known that Josephson junctions can be used as a TeraHertz source of electromagnit radiation. This TeraHertz radiation is in between the infrared and microwave elctromagnetic waves and can penetrate the objects and materials which are opaque for the visible light. TeraHertz waves can be applied in numerous areas of scientific knowledge, such as a chemistry analysis, security screening, detection of hazardous materials from distance, non-destructive determination cancer tumors on the human body, wide-band secure communications, etc. Having in mind the communication applications, on has to keep in mind that the Earth atmosphere absorbs TeraHertz waves strongly, that is why such waves can be used in satellite-to-satellite, satellites to plane communications over long distances above the Earth atmosphere. For short distances (several tens of meters) these waves can be used for wide- band wi-fi systems.

As for the Josephson junctions as a TeraHertz radiation sources, it should be noted that a single Josephson junction generates a pico-, nano-Watt power, which is not enough for the practical applications. For these reasons synchronization of a larger number of Josephson junctions could be a way out of this difficulty. It is well-known that a power generated by a coherent wave sources is proportional to the square of the source numbers.This phenomenon is called a superradiattion. Fortunately, it was established recently that many of high-temperature superconducting materials contain hundreds, thousands Josephson junctions. Synchronization of these junctions can be quite heplful to achieve a power around milli-Watt, which is quite acceptable from the practical application point of view. It should also be noted that if necessary such junctions can be greated artifically. Therefore synchronization of large amount of Josephson junctions is an important subject of research in this dissertation.

For thus purpose Chapter VI deals with the approach how to synchronize large amount of the Josephson junctions.The case considered in Chapter VI corresponds to chaotic Josephson junctions, although the results are also valid for the case of non-chaotic dynamic of these junctions.

Thus in Chapter VI the study of synchronization between the Josephson junctions is the aim of research. In considered topology the driving junction is coupled to the junction in series. The coupling occurs with some delay, and couping is unidirectional. It has been established that there is high quality synchronization between Josephson junction in such a configuration. Most importantly numerical simulations shows that in series coupled Josephson junctions a number of junctions synchronized is not high, around 10 junctions. It is also shown that the number of synchronized junctions can be increased slightly if the distance between junctions is decreased. It is obvious there is a limit to such a reduction. This results underline that other coupling topologies are to be considered to increase the number of synchronized junctions drastically. The result of Chapter VI is published in paper [9].

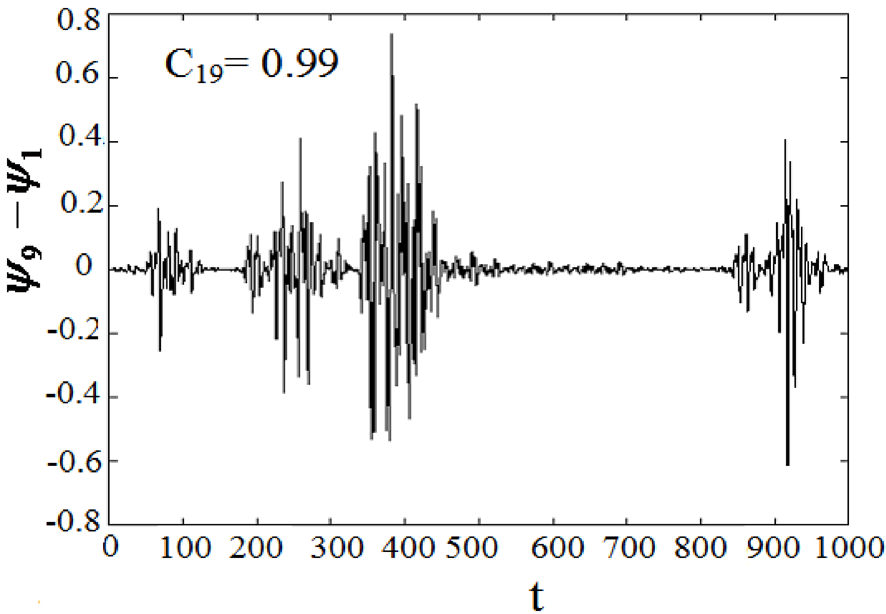

Figure 7. Synchronization between 9 Josephson junctions: synchronization eror dynamics between the 1-st and 9–th junction is shown.

$C_{19} = 0.99$ is the correlation coefficient between the junctions. ψ is the phase difference dynamics with time between the corresponding junctions.

RESULTS

This dissertation deals with chaos synchronization between functional feedback(s) systems. The synchronization of chaotic dynamics between transmitter and receiver systems is important for message decoding at the receiver end. In this dissertation work models under study are famous in non-linear dynamics Ikeda, Mackey-Glass and Josephson systems.

Synchronization in complex systems are important for computer modelling of such systems. Synchronization phemonen can be used for creating dynamic logic elements, which morphe (convert) into each other could eventually enhance computer processing speed of information.

The main results of this dissertation are:

1. Existence and stability conditions for the synchronization regimes are found for synchronization between feedback functional systems.Synchronization quality is nearly unaffected under parameter mismatches 1-5 %.
2. It is found that under certain conditions parameter mismatches are required for synchronization. Stability conditions chaos synchronization are found for the case of modulated feedback.
3. Stability conditions for synchronization regimes between nonlinear systems with several feedbacks are determined with the help of Lyapunov - Razumikhin functional approach. The number of synchronization regimes for systems with several feedbacks is higher in comparison with systems with a single feedback. This result is of certain importance from the practical point view.
4. Conditions for the generalized synchronization under parameter mismatches are found for Mackey-Glass and Ikeda systems and on this basis further generalization for this type of nonlinear systems is worked out.
5. It is established that synchronization between several Josephson junctions coupled in series and uni-directionally is possible.

In this work study of anticipating synchronization occupies an important place. During this process slave system is synchronized to the future state of the master system, i.e. it feels the future of master system. This property is very important from the application point of view. To be more concrete this property is vital for quick prognosis tasks, as the only thing to do is connect identical slave system to the master system. Therefore anticipating synchronization can be used for non-invasive diagnostic problems, in the information processing purposes in the natural systems, etc. Finally in chaos based communication systems anticipating synchronization can provide additional time for the unauthorised intruder for secret message deciphering -not desirable perspective.

Selective synchronization is another important part of this work. By choosing parameter mismatches selectively one can exclude some types of synchronization.-an important property from the application viewpoint, especially in the information processing tasks, including natural systems.

In this work due attention is also given to chaos synchronization between multi-feedback systems. Additional feedbacks can be used for the stabilization of laser intensity and for an adequate reaction to the external influences. As the multi-feedback systems can provide higher and larger number of positive Lyapunov exponents such systems could provide more security in chaos based communication schemes.

One of the important research subjects in the dissertation work is the investigation of generalised synchronization between functional systems. In this case there is a functional dependence between the states of the synchronized systems. This property makes generalised synchronization more suitable for the chaos based secure communication systems, as unauthorised third party should figure out the the form of the functional dependence in order to desipher the masked with chaos message.

The study of chaos synchronization between Josephson junctions is one of important researh topics in the dissertation work. Such synchronization can provide terahertz wave sources with adequate power for practical applications. The case considered in this work deals with the Josephson junctions coupled undirectionally in series. Results of this work show in such a configuration synchronization of a large number of junctions (thousands, tens of thousands) is problematic. Conclusion is that other connection topologies, such as Josephson junctions connected in paralell should be studied. Current terahertz sources are of large dimention, costly, not mobile etc. These sources include laser installations, synchrotron sources, free electron lasers, etc. Teraherz sources based on the synchronization between thousands or more Josephson junctions could be cost effective, mobile, with small sizes, etc. This property makes them more amenable for the application purposes. Terahertz waves can be used for the detection of hazardous materials from distance, for the sreening porposes, detection of plastic mines, in ecology, for non-invasive diagnostics of skin cancers, wi-fi systems etc.

Results of the dissertation are published in the following papers and contributions:

1. Shahverdiev, E. Anticipating chaos synchronization in time – delayed systems/ E.Shahverdiev, R.Nuriev, G. Gasimova [et al.] // Fizika (Azerbaijan), –2002, 8(3), – pp.9-11.
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