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ABSTRACT

of the dissertation for the degree of Philosophy

THE SPECTRAL PROPERTİES OF THE WEIGHTED ENDOMORPHISMS OF UNIFORM ALGEBRAS

Speciality: 1202.01- Analysis and functional analysis

Field of science: Mathematics

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GENERAL CHARACTERISTICS OF THE WORK

Rationale of the topic and development degree. Since most of the processes that maintain the internal structure of space are usually in the form of composition operators and correspond to endomorphisms in regular algebras, the study of their spectral properties, as well as weighted endomorphisms, is important. Since the middle of the last century, especially after the Kamovic-Scheinberg[1] theorem, research in this field has expanded in various algebras and is actively continuing in modern times. Clearly, the study of the spectrum of linearly limited operators acting in Banach spaces, as well as their spectral properties, has always been a topical issue .One of the reaserchs considered from this point of view is the theorem that the spectra of nonperiodic automorphisms of H. Kamovic and Scheinberg's semisimple Banach algebras contain a unit circle with a central coordinate system. One of the reaserchs considered from this point of view is the theorem that the spectra of non-periodic automorphisms of H. Kamovic and Scheinberg's semisimple Banach algebras contain a unit circle which mapping or keeping itself with a central coordinate system. Since such algebras could be realized in the form of uniformed algebras of continuous functions defined in certain compacts by Gelfand's theory, their endomorphisms (as well as the more common weighted composition operators that affect their closed subspaces)began to be studied in regular spaces of compact continuous functions, especially in uniformed algebras. It is true that the compositional operators, which are a special case of the general operators we have mentioned, in a number of periods, mainly in the analytical structure, independent of the Kamovich-Scheinberg theorem in function spaces were studied from the point of view of spectral properties.In this direction R.B.Monatodor, A.K.Kitover, R.N.Levin, A.B.Antonevich, V.G.Kurbatov, S.Onho, J.Vaden and others their research can be

[1] Kamowitz H.,Scheinberg S. The spectrum of automorphisms of Banach algebras //-New York: Journal of Functonal Analysis,-1969.v.4,№2,-p.268-276

noted. However, in all these works, mainly endomorphisms, weighted endomorphisms have been studied in specific algebras. A more common case of such weighted endomorphisms, weighted composition operators began to be studied from the point of view of compactness in the uniform closed subspaces, regular subalgebras of the $C(X)$ space of all continuous functions defined in the X compact by A.I.Shahbazov[2] and E.A.Gorin[3] since 1980. Thus, in the work of A.I. Shahbazov, the terms of the set of peaks defined for regular algebras, the terms of peak points were generalized for regular closed subspaces as well. These terms with the help of the above mentioned in those uniformly closed spaces compactness conditions were obtained for weighted composition operators and their weighted in their special uniform algebras application for the compactness of endomorphisms are.

For the compactness of weighted endomorphisms in the dissertation generalizations have been assigned. As well as in the dissertation uniform space of continuous functions defined in the compact and weighted, which affects its uniform closed subspaces composition operators and also in special cases, uniformly algebraic weights, other spectral properties of weighted endomorphisms, such as nucleation, closedness of range, and Hyer-Ulam stability have also have been researched.In this work investigated the eigenvalues and eigenspaces of algebra of convergent power series and related theorems have been proved.

All this indicates the relevance of the dissertation topic.

Object and subject of research. The object of the dissertation is the weighted and weighted type endomorphisms of uniform algebras, and the subject is the spectral properties of these endomorphisms.

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[2] Шахбазов А.И. О некоторых компактных операторах в равномерных пространствах непрерывных функций // –Баку: Доклады АНА ССР,– 1980, т.36, № 12, –с.6-8.

[3] Горин Е.А., Шахбазов А.И. Компактные комбинации взвешенных подстоновок диск-алгебры // –Воронеж: Труды XVII Воронежской Зимней матем.школы, –1983, –с.69-71.

Objectives and tasks of the research

-Obtaining the necessary and sufficient conditions for the compactness of the weighted composition operator, which affects the space of continuous functions in the compact.

-Determination of the criteria of compactness of weighted endomorphisms of uniform algebras with analytical structure;

-Acquisition of necessary and sufficient conditions for the stability of the non-trivial weighted composition operator in the Hyer-Ulam sense;

-Determination of eigenvalues and corresponding eigensubspaces of endomorphisms of the algebra of convergent power series;

-Determination of the relationship between the eigenvalues of the weighted endomorphism in the uniform algebras with analytical structure and the eigenvalues of the endomorphism of the algebra of the convergent power series with the corresponding eigensubspaces;

Research methods. The dissertation is based on the general ideas of functional analysis and algebra. The methods of commutative Banach algebras given on the complex field, uniform algebras with its special case and the theory of analytical functions were used.

The main thesis to be defended. The uniform space of continuous functions given in a certain compact and the issues of compactness, nuclearty and closedness of the range of weighted composition operators in the uniform closed subspaces of this space are studed. The obtained results are also applied to the study of spectral properties of weighted endomorphisms when the closed subspaces under consideration are uniform algebras with an algebraic structure.

The general eigenvalues and eigensubspases of the endomorphisms of algebra of the convergent power series are researched.The eigenvalues of the nonresonancing endomorphism of algebra $\sum_n$ are determined on the basis of the eigenvalues of the mapping indused by that and the sizes of the corresponding eigensubspases are determined. Also, resonancing endomorphisms, which are resonancing monoms of algebra of the convergent power series $\sum_2$, resonancing endomorphisms of nonresonancing monoms and the corresponding eigensubspaces dimensions are assigned by calculating the eigennumber of nonresonancing endomorphisms.

In uniform algebras with an analytical structure, the weighted endomorphism caused by the Dencoy-Wolff fixed point, which transforms the compact to which the algebra is assigned, is self-evident relationship between the eigenvalues and the corresponding eigensubspaces and the eigenvalues of the endomorphisms of algebra of the convergent power series and the corresponding eigensubspaces is studied.

Scientific novelty of the study. Scientific novelty of the research. It is a criterion for the spectral properties of the continuous space of continuous functions defined in the dissertation and the compactness, nucleus, closedness of the range and Hyer-Ulam stability of weighted composition operators (especially in the case of weighted and weighted type endomorphisms of uniform algebras) acting on its uniform closed subspaces. In the algebra of the convergent power series, the eigenvalues of these types of operators and the corresponding eigensubspaces are described with biholomorphism accuracy. In general, the results obtained for the algebra under consideration are shown in the ways of application of compact weight endomorphisms affecting the analytical structural algebras to eigensubspaces.

Theoretical and practical value of the study. Theoretical and practical significance of the research. The results obtained in the dissertation are mainly theoretical. The results obtained in this work can be applied to the solution of homogeneous and non-homogeneous functional (intego-functional) equations in a compact.

Approbation and application. The main scientific results of the dissertation were presented at the following republican and International conferences, as well as reports and discussions at scientific seminars: In scientific seminars of the department of "General Mathematics" (supervisor-prof. M.Sh.Hajiyev) of Nakhchivan State University; In scientific seminars of the department of “Higher mathematics and informatics” (supervisor-doc. S.A Aliyev) of Nakhchivan Teachers' Institute; At the scientific seminar of the departments of the Institute of Mathematics and Mechanics of NASA "Non-harmonic analysis" (supervisor-corr.-member of the NASA, doctor. of phys.- math. sc., prof.B. Bilalov), “Functional analysis” (supervisor-doctor. of phys.- math. sc., prof. H.I. Aslanov) and "Algebra and mathematical logic"(supervisor-doc. A.A.Babayev); At the International Conference on Physics, Mathematics and Technical Sciences (Nakhchivan 2008);At the International Conference dedicated to the 50th anniversary of the Institute of Mathematics and Mechanics of ANAS (Baku 2009);International conference on "Spectral theory and its applications" dedicated to the 80th anniversary of Academician F.G Maksudov (Baku 2010);At the International Conference dedicated to the 100th anniversary of Academician ZI Khalilov (Baku 2011); At the Republican Scientific Conference "Actual Problems of Mathematics and Mechanics" dedicated to the 90th anniversary of the national leader of Azerbaijan Heydar Aliyev (Baku 2013);At the Fourth International Scientific Conference "Mathematical Innovations and Applications" (Istanbul 2021).

Author’s personal contribution. The obtained results and suggestion belong to the author.

Author’s publications. Author's publications. The main results of the disertation appear in 2 journals of the applicant Zentralblatt MATH are reflected in 9 scientific articles published in scientific publications recommended by SAC.Two of these articles are not co-authored.In addition, the results of the disertation were presented in 7 international and 1 national scientific conferences and these reports were reflected in the relevant conference materials as theses.One of them was published abroad.

The organization where the dissertation work was done. Name of the organization where the dissertation work is performed.The dissertation work was carried out at the Department of General Mathematics of Nakhchivan State University.

Total volume of the work. The total volume of the dissertation with a sign, indicating the volume of the structural units of the dissertation separately.

The dissertation consists of an introduction - 48528 characters, title page and table of contents - 2611 characters, three chapters - 143091 characters (Chapter I- 88000, Chapter II- 40599, Chapter III-14492), results - 1666, a list of 86 titles. The total volume of the dissertation is 195896 characters.

THE CONTENT OF THE DISSERTATION

The dissertation work consists of an introduction and three chapters. The introduction substantiates the relevance of the topic of the dissertation and gives a brief analysis of the work done.

The first chapter examines the spectral properties of the weighted composition operators of the space of continuous functions defined in the compact. First, the basic concepts for this chapter are included.

Definition 1. A closed subset $E \subset X$ is called a peak set with respect to $A(X)$, if there exists a sequence $\{f_n\} \subset A(X)$, such that $\|f_n\| = f_n(x) = 1$ for all n and all $x \in E$, moreover, outside any neighbourhood of the subset E the sequence $\{f_n\}$ tends to zero uniformly. A peak set consisting of only one point is called a peak point

In the work denoted the set of all peak sets with respect to $A(X)$ by $S(A(X))$ and the set of all peak points with respect to $A(X)$ by $S_0(A(X))$.

Definition 2. Let $A(X)$ be uniformly closed subspaces of $C(X)$ (in particular, a uniform algebra). A mapping $\varphi : X \to X$ is called a compositor on $A(X)$ if $f \circ \varphi \in A(X)$ whenever $f \in A(X)$. A function $u \in C(X)$ is called a multiplicator whit respect to $A(X)$ if $u \cdot f \in A(X)$ for all functions $f \in A(X)$.

We denot the set of all compositors on $A(X)$ by $C_{A(X)}$ and the set of all multiplicators whit respect to $A(X)$ by $M_{A(X)}$.

Definition 3. If $Tf \in A(X)$ for every $f \in A(X)$, than a mapping $\varphi : X \to X$ is called a composition with respect to weighted composition operator of the form $T : A(X) \to C(X)$, $f \mapsto u \cdot f \circ \varphi$, $u \in M_{A(X)}$ and subspaces $A(X)$. In this kind case will be denoted by $\varphi \in C_{T,A(X)}$.

In 1.1, for the uniform closed subpaces of the sup-norm spaces of the continuous functions $C(X)$ defined in a given X metric compact with the help of the term peak points the necessary condition for compactness of weighted composition operators induced by compact non-continuous maps.The following definitions are given which are necessary to solve this problem.

Theorem 1. If $u \in M_{A(X)}$ and $\varphi \in C_{T,A(X)}$, then the weighted composition operator of the form $f \mapsto u \cdot f \circ \varphi$ (where $u \in C(X)$ is a fixed function and φ is a selfmapping of X whichis continuous on the support of function u,i.e., on the open set $S(u) = \{x \in X | u(x) \neq 0\}$) is compact if and only if, for evry compactly connected componenet K of $S(u)$ and for any peak set E with respect to $A(X)$, we have either $\varphi(K) \bigcap E = \varnothing$ or $\varphi(K) \subset E$.

The application of the above theorem, when the uniform closed subspace $A(X)$ is a uniform subalgebra of the space $C(X)$, leads the problem under consideration to the criteria of compactness of the weighted composition operators of this algebra in that algebra. For example, the following theorem is obtained for the compactness of the weighted composition operator of the algebra $C(X)$, which is a universal uniform algebra in a given X compact.

Theorem 2. The weigheted composition operators T on uniform algebra $C(X)$ given on a compact X is compact if and only if, for every compact subset $K \subset S(u)$ we have $\varphi(K)$ finite set.

The same problem has been studied and the following theorem have been obtained since X is a locally connected compact.

Theorem 3. Let X be a locally connected compact and each operator in the form $f \mapsto f \circ \varphi$ is compact on the $P \subset X \setminus S_0(A(X))$ compact set. The weighted composition operator T induced by $u \in M_{A(X)}$ and $\varphi \in C_{T,A(X)}$ is compact if and only if, when for every connected compact component $K \subset S(u)$ either $\varphi(K)$ is one-point set, or $\varphi(K) \subseteq X \setminus S_0(A(X))$.

The following theorem is proved for related compacts..The $\dot{I}ntX = X^0$ inner part of the X compact is not empty. At the same time, there is a dense set everywhere in the compact. Then the following sentence is true for uniform closed subspaces $A(X) \subset C(X)$ which can be obtained from the sequence of locally limited functions in the inner part X^o that the sequence is a normal family.

Theorem 4. Let X be a connected compact and $S_0(A(X)) = \partial X$ (here ∂X is topological boundary of X). The nontrivial weigheted composition operators T induced by $u \in M_{A(X)}$ and $\varphi \in C_{T,A(X)}$ is compact if and only if, for every connected compact component $K \subset S(u)$ such that $\varphi(K) \subset X \setminus S_0(A(X))$.

In 1.2, when the subspace $A(X)$ is a uniform algebra, the results related to the compactness and nucleation of its weighted endomorphisms are obtained. Thus, the following theorem is obtained, which generalizes the criterion of compactness for nontrivial weighted endomorphisms in the case of disc-algebra $A(D)$ (ie, the uniform algebra of functions analytic on the open unit disc $D = \{z \in C : |z| < 1\}$ of the complex plane C and continuous on its closure) in uniform algebras with other analytical structure

Theorem 5. $A(X)$ uniform algebra with analytic structure. If $u \in M_{A(X)}$ and $\varphi \in C_{T,A(X)}$, then the nontrivial weighted endomorphism T is compact if and only if, for every compactly connected component X we have either φ is constant mapping, or $\varphi(x) \in X \setminus S_0(A(X))$ whenever $x \in S(u)$.

According to Theorem 5, we can look at ball and polydisc algebra, which are multidimensional cases of disc-algebra. Results were obtained that determined the compactness of the weighted endomorphisms created by the maps, which at some point lost their analyticity on these algebras.

Suppose that $A(B^n)$ is a ball algebra, that is a uniform algebra of analytic functions with in the uniform space $B^n = \left\{ z = (z_1, z_2, ..., z_n) \in C^n : \sum_{k=1}^{n} |z_k|^2 < 1 \right\}$ of the n-dimensional C^n complex space, and of continuous functions on its closure. In this case, since each point on the topological boundary of the ball is the peak of the ball algebra, the following theorem is true for the compactness of the weighted endomorphism discussed above.

Theorem 6. The weighted endomorphism induced by u and φ on $A(B^n)$ (u and φ are analytic) is compact if, and only if ,either $\varphi = conct$, or $\|\varphi(z)\| < 1$ (Eucledian norm) for all $z \in S(u)$.

This section also studed weighted endomorphisms of polydisc algebra $A(D^n)$. Let $D^n = \{z = (z_1, z_2, ..., z_n) \in C^n : |z_k| < 1, 1 \le k \le n\}$, $A(D^n)$ is the algebra of analytic functions in D^n and continuous on its closur. The Shilov's boundary of $A(D^n)$ is the torus $T^n = \{z = (z_1, z_2, ..., z_n) \in C^n : |z_k| = 1, 1 \le k \le n\}$ and it is contained in the topological boundary ∂D^n as a proper subset for $n > 1$

Theorem 7. The endomorphism $T : A(D^n) \to A(D^n)$, of weighted composition induced by $u \in M_{A(X)}$ and $\varphi \in C_{T, A(X)}$ $(\varphi : D^n \to D^n, \varphi = (\varphi_1, \varphi_2, ..., \varphi_n))$ on $A(D^n)$ is compact if only if, either for any $1 \le k \le n$ $\varphi_k(z) = const$ or $|\varphi_k(z)| < 1$ for all $z \in S(u)$.

İn 1.2 examined the nuclearity properties of weighted endomorphisms in uniform algebras. Suppose that the set $X \subset C$ is a compact containing a closed unit disk $\bar{D}$ of the complex plane C and $A(X)$ is a uniform algebra with a set of peak points $S_0(A(X))$ defined in that compact. Here the condition $X \setminus S_0(A(X)) = D$ is satisfied and the functions of the algebra $A(X)$ are analytic in the open D domain. Under these conditions, the nontrivial weighted endomorphism T of algebra $A(X)$ ($u \in M_{A(X)}$ və and $\varphi \in C_{T,A(X)}$) is nuclearted beyond the set $S(u)$, even in relation to the mapping φ, which have lost their continuity, which expresses the necessary and sufficient condition in a generalized form, has been proved in this following theorem.

Theorem 8. Let $A(X)$ be a uniform algebra on compact X If $u \in A(X)$ is zero on the set $S_0(A(X)) \setminus \partial D$, then the nontrivial weighted endomorphism T is nuclear if and only if, there exists a constant $M > 0$ such that $|u(z)| \leq M(1-|\varphi(z)|)$ for all $x \in X$.

İn 1.3 examines the closedness of range and Hyer-Ulam stability of weighted endomorphisms in uniform algebras. Thus, in this section, the following criteria are obtained for the closedness of range of nontrivial weighted composition operators in uniform closed subpaces $A(X)$ within certain conditions imposed on the peak sets, peak points of the uniform algebra $C(X)$ defined in compact X , as well as the topological boundaries of the compact.

Theorem 9. If $u \in M_{A(X)}$ and $\varphi \in C_{T,A(X)}$, then the nontrivial weighted composition operator T has closed range if and only if, there exists a compact subset K of $S_0(A(X)) \bigcap S(u)$ such that $S_0(A(X)) \subset \varphi(K)$.

Under the conditions of Theorem 9, the set of peak points $S_0(A(X))$ coincides with the topological boundary of the compact where the set is determined and having a functional boundary with respect to a given subspace closedness of the range of weighted composition operator affecting the regular closed subpaces of $A(X)$ obtained as follows in this criterion obtained as follows.

Theorem 10. If $u \in M_{A(X)}$ and $\varphi \in C_{T,A(X)}$, then the nontrivial weighted composition operator T has closed range if and only if, there exists a compact subset K of $\partial X \bigcap S(u)$ such that $\partial X \subset \varphi(K)$.

From this theorem we obtain the necessary and sufficient conditions that can be easily verified for the closedness of ranges of weighted endomorphisms acting on ball and polydisc algebras with an analytical structure, coinciding with the topological boundary of the compact where the set of peak points is determined.

Corollary 1. The nontrivial weighted endomorphism induced by u and φ on $A(B^n)$ (u and φ are analytic) has closed range if and only if, there exists a constant $\delta > 0$ such that $\varphi(\{z \in \partial B^n : |u(z)| \geq \delta\})$ contains the boundary ∂B^n of the unit ball B^n.

Corollary 2. The nontrivial weighted T endomorphism induced by u and φ on polydisc algebra $A(D^n)$ (u and φ are analytic) has closed range if and only if, there exists a constant $\delta > 0$ such that $\varphi(\{z \in T^n : |u(z)| \geq \delta\})$ contains of T^n

In 1.3 also examines the Hyer-Ulam sense stability of a nontrivial weighted composition operator. Hyer-Ulam stability for the $T : A(X) \to A(X)$ operator was first studied in the general case, even if it does not have of uniform closed $A(X)$ subspaces specific structures (eg, algebraic, analytical, etc.). In the next stage, with the help of the obtained results, the Hyer-Ulam stability criteria were obtained for the operators considered in cases where the subspace $A(X)$ has the above mentioned special structures.

Theorem 11. If $u \in M_{A(X)}$ and $\varphi \in C_{A(X)}$, then the nontrivial weighted composition operator T has the Hyer-Ulam stability if and only if there exists a compact subset Y of $S(u)$ such that $K(T) \cap S(A(X)) \subset \varphi(Y)$.

Here, the closure of $\varphi(S(u))$ in relation to the topology of space is the set $K(T)$ and $S(A(X))$ is the Shilov boundary.

The following result is obtained for the Hyer-Ulam sense stability of T nontrivial weighted endomorphisms of uniform algebra $C(X)$.

Corollary 3. The nontrivial weighted endomorphism has the Hyers-Ulam stability if and only if, there exists a compact subset K of *S(u)* such that $\varphi(K) = \varphi(S(u))$.

From this corollary we have easily verifiable necessary and sufficient conditions for the weighted composition operators on the classical uniform algebras which have analytically structure. In case topological boundary of compact coincide with Shilov's boundary such as in the ball algebra $A(B^n)$ we have following results.

Corollary 4. The nontrivial endomorphism of weighted composition induced by *u*, φ on $A(B^n)$ (*u* and φ are analytic) has the Hyers-Ulam stability if and only if, there exists a constant $\delta > 0$ such that $\varphi(\{z \in \partial B^n : |u(z)| \geq \delta\})$ contains the boundary ∂B^n of the unit ball B^n.

Corollary 5. The endomorphism $f \mapsto f \circ \varphi$ on the ball algebra $A(B^n)$ induced by a nonconstant analyitic mapping $\varphi : B^n \to B^n, (n \geq 1)$ has the Hyers-Ulam stability if and only if, the set $\varphi(\partial B^n)$ contains the boundary ∂B^n of the unit ball B^n.

In 1.4 examines the conditions of their compactness as spectral properties of weighted type composition operators.Thus, the following lemma is given, which allows us to study the compactness properties of weighted composition operators, as well as their finite sums called weighted type composition operators, in the $C(X)$ space given in compact and in the uniform closed $A(X)$ subspaces of $C(X)$ space. This is mainly due to the issues of assembly in the dual Banach spaces.

Lemma1. Weighted composition type operator $T_1 : A(X) \to C(X),\ f \mapsto \sum_{i=1}^{n} u_i \cdot f \circ \varphi_i$ on $A(X)$ is compact if only if,

$\sum_{i=1}^{n} u_i(x)\cdot\left[\delta_{\varphi_i(\xi)}-\delta_{\varphi_i(x)}\right]\to 0$ when $\xi\to x$ with respect to $A(X)^*$-norm. (here δ_x, $\delta_x(f)=f(x), x\in X$ is Dirak functional on $A(X)$ and $A(X)^*$ is dual $A(X)$).

With the help of Lemma 1, the following definitions are given to prove the theorem that determines the compactness of nontrivial weighted type composition operators in space $C(X)$, not only in metric compacts, but also in arbitrary X topological Hausdorff compact space. The following definitions are given for this. Let $x\in S_0(A(X))$ be fixed.

Definition 4. Let $x\in S_0(A(X))$ be a fixed point, we say that indices $i, j\in\{1,2,...,n\}$ (for indexes in defined the T_1 operator) are equivalent with respect to $x\in S_0(A(X))$ if $\varphi_i(x)=\varphi_j(x)\in S_0(A(X))$. Equivalent classes of this kind will be denoted by *K*.

Definition 5. Indices $i, j\in\{1,2,...,n\}$ (for indexes in constructing defined the T_1 operator) are called strongly equivalent if $\left\|\varphi_i(y)-\varphi_j(y)\right\|_{A(X)^*}\to 0$ when $y\to x$, then the then the indices i and j are strongly equivalent to point X.

In the strongest sense, we will denote the equivalence classes by L. Clearly, the $L\subset K$ condition is paid for each L class. The following criterion is obtained for the compactness of weighted type composition operators in case $A(X)=C(X)$ for a certain L equivalence class.

Theorem 12. The nontrivial wighted composition type operator $T_1: C(X)\to C(X),\ f\mapsto \sum_{i=1}^{n} u_i\cdot f\circ\varphi_i$ on $C(X)$ is compact, if only if, for every $x\in X, u_i(x)\neq 0, i\in\{1,2,...,n\}$ ether $\varphi_i(\xi)=\varphi_j(\xi), \xi\in U$, $\sum_{j\in L} u_j(x)=0$ or $\varphi_i(\xi)=\varphi_j(\xi), \xi\in U$,

$\sum_{j\in L} u_j(x) \neq 0$ with respect to indices $i, j \in L$ (where U is neighborhood of $x \in X$).

The second chapter considers the general eigenvalues and eigensubspaces of the endomorphisms of the algebra $\sum_n$ of the convergent power series of $z = (z_1, z_2, ..., z_n)$ n-variables. Then, the spectral properties of the endomorphisms of the algebra $\sum_2$ in the case of $n = 2$ are investigated.

In 2.1 the eigenvalues and eigensubspaces of the nonresonancing endomorphisms of the algebra of the convergent power series are investigated. In this chapter, the concept of resonance, as well as the concept of resonancing monoms in the case of resonance are given as follows.

Definition 6. $\alpha_1, \alpha_2, ..., \alpha_n$ are eigenvalues of mapping φ of its linear part φ_1. If $\alpha_s = \alpha^m = \alpha_1^{m_1} \cdot \alpha_2^{m_2} \cdot ... \cdot \alpha_n^{m_n}$ (here $m_i \geq 0, \sum_{i=1}^{n} m_i \geq 2$), then $\alpha_s, 1 \leq s \leq n$ is called resonancing eigenvalue; for any resonancing eigenvalue $\alpha_s = \alpha^m$ correspoinding resonancing vector-monom $z^m e_s$. Here e_s is the basic vector and which is denoted by $z^m = z_1^{m_1} z_2^{m_2} \cdot ... \cdot z_n^{m_n}$

Definition 7. If between eigenvalues $\alpha_1, \alpha_2, ..., \alpha_n$ there is resonancing condition, then the endomorphism induced by φ is called resonancing endomorphism, otherwise is nonresonancing endomorphism.

Theorem 13. If modules of eigennambers $\alpha_1, \alpha_2, ..., \alpha_n$ of the linear part of mapping φ wich generated the endomorphism $T : \sum_n \to \sum_n$ are less than one and nonzero, nonresonancing, differently , then eigenvalues of T have the form $\lambda_k = \alpha_1^{k_1} ... \alpha_n^{k_n}$, where $k_i \in Z_+, i = 1, 2, ..., n$ and corresponding eigensubspaces up to diffeomorphism are generated by the functions $f_k = z^k$ Consequently, all eigensubspaces are one dimensional (here the multi-index $k=(k_1,k_2,...,k_n)$ is defined as $a_k = a_{k_1,k_2,...,k_n} \neq 0$ in the series $f(z)=\sum_k a_k z^k \in \Sigma_n$).

In 2.2 defines the eigennambers of resonant endomorphisms of resonancing monoms of algebra Σ_2 and descriptions of eigensubspaces are given, as well as their dimensions are calculated.So here in the algebra Σ_2 of series (formal or convergent series) of the form

$$\sum_{n,m\geq 0} a_{n,m} x^n y^m \tag{1}$$

and spectral properties of endomorphism T of this algebra induced by map φ, which module of eigenvalues of linear part of φ are less than one have been studied. That is, the eigenvalues and eigensubspaces of the form $T:\Sigma_2 \to \Sigma_2$, $f \to f \circ \varphi (f \in \Sigma_2)$ endomorphism of algebra Σ_2 have been investigated.Which is α_1, α_2 are eigenvalues of the lienar part of mapping φ. This numers satisfies the condition $\alpha_i : 0 < |\alpha_i| < 1 (i=1,2)$ and $\alpha_1 = \alpha^m, \alpha_2 = \alpha$ (here m is an order of resonancing conditions) The following theorem is proved here.

Theorem 14. In the resonancing cases with the resonancing monoms every eigenvalue of endomorphism $T:\Sigma_n \to \Sigma_n$ has the form $\lambda_q = \alpha^q$ (here q is nonnegative whole number) and corresponding eigenfunction has the form $f_q(x,y)=y^q$ (or has the form $f_q(x,y)=x^q$). So the corresponding eigensubspaces are one-dimensional.

In 2.3, the eigenvalues and eigensubspaces of the resonancing endomorphisms without the resonancing monom of the problem are studied in (special case of 2.2)

Theorem 15. In the resonancing cases without resonancing monom every eigenvalue of endomorphism $T:\sum_2 \to \sum_2$ has the form $\lambda_q = \alpha^q$ (here $q > m$, m is an order of resonancing conditions) and corresponding eigenfunctions have the forms

$$f(x,y) = \sum_{k=0}^{\left[\frac{q}{m}\right]} a_{k,q-mk} x^k y^{q-mk} \text{ or } f(x,y) = \sum_{k=0}^{\left[\frac{q}{m}\right]} a_{k,q-mk} y^k x^{q-mk} \qquad (2)$$

Consequently, corresponding eigensubspaces are $(\left[\frac{q}{m}\right]+1)$-dimensional.

In 2.4, examines the eigenvalues and eigensubspaces of nonresonancing endomorphisms of the algebra $\sum_2$. The following theorems are proved here according to the multiplicative dependence of eigenvalues.

Teorem 16. If there are nonresonance and nonmultiplicativ dependencies between the eigenvalues α_1, α_2 then each eigenvalue of the endomorphism $T:\sum_2 \to \sum_2$ is in the form $\lambda_{(k,l)} = \alpha_1^k \alpha_2^l$ (here $k > 0,\ l > 0$ is $\alpha_1^k \cdot \alpha_2^l \neq 1$) and the eigenfunctions corresponding to this eigenvalue is in the form $f(x,y) = x^k y^l$. Thus, we obtain that the corresponding eigensubspace are one-dimensional.

Theorem 17. If between eigenvalues α_1, α_2 of the linear part of mapping φ there is not resonancing conditions, but they are multiplicative depending, then every eigenvalue of the operator $T:\sum_2 \to \sum_2$ has the form $\lambda_{(k,l)} = \alpha_1^k \alpha_2^l$ for some $(k,l) \in Z_+ \times Z_+$ and corresponding eigensubspaces consists of polynoms

$$f(x,y) = \sum_{\substack{s \in Z \\ k+sqm_2 \geq 0 \\ l-sqm_1 \geq 0}} a_s x^{k+sqm_2} y^{l-sqm_1} \qquad (3)$$

Therefore corresponding eigensubspaces are finite dimensional and dimensional is

$$dim\, E_T(\alpha_1^k \alpha_2^l) = \left| Z \cap \left[-\frac{k}{qm_2}, \frac{l}{qm_1} \right] \right| \qquad (4)$$

(where the symbol $|A|$-defined power of the set A and $\alpha_1^a \alpha_2^b = 1, a = sqm_2, b = -sqm_1, a \neq 0, b \neq 0$ və $q, m_1, m_2 \in Z_+$).

In the third chapter in 3.1 the relation between eigennambers and corresponding eigenfunctions of endomorphisms C_φ induced by selfmap $\varphi : K \to K$, which has a Dencoy-Wolf type fixed point (on the algebra $A(K)$ which has analitical sturucture) and $\left[C_\varphi\right]_0$ on the algebras of convergent power series $\sum_n$ of n-variables $z = (z_1, z_2, ..., z_n)$, i.e.,on the algebras $O_0(D)$ of germs of the functions of $A(K)$ at the point zero (for simply we assume the point zero is a Dencoy-Wolff type fixed point and we consider the case $u \equiv 1$) have been investigated. Here $\left[C_\varphi\right]_0 : \sum_n \to \sum_n, f \mapsto f \circ \bar{\varphi}_0$ and $\bar{\varphi}_0$ is normal form of map φ.

Theorem 18. If a matrix of linear part A of φ at the point zero is diagonalizable, then eigennumbers of compact operator C_φ and $\left[C_\varphi\right]_0$ are consides and for every nonzero eigennumber $\mu \neq 0$, there is a biholomorphic isomorphism between eigensubspaces $L_\mu(C_\varphi)$ and $L_\mu(\left[C_\varphi\right]_0)$.

In 3.2, the weighted composition operator (as well as the generalization of endomorphism to weighted endomorphism), which is a generalization of the composition operator of a uniform algebra with an analytical structure, more precisely, the forms

$$Tf(z) = u(z) \cdot (C_\varphi f)(z) = u(z) \cdot f(\varphi(z)), (f \in A(K)) \qquad (5)$$

like $T: A(K) \to A(K)$ weighted composition operator considered. Here $u \in A(K)$ is the fixed function and $\varphi: K \to K$ is a fixed continuous self-mapping, which is analytic in $\dot{I}nt\,K$ of the compact. Also zero point is the Dencoy-Wolff type fixed point of the φ map. Accordingly, the following theorem has been proved.

Theorem 19. Between eigennumbers (and also, corresponding eigenfunctions, i.e., eigensubspaces) of operators T and T_0 (here $T_0 : \sum_n \to \sum_n$, is the operator created by the germ of the weight function in algebra $\sum_n$) there is a bijective relation.

CONCLUSION

The following new scientific results were obtained in the dissertation:

The compactness criteria of this operator have been determined in the case of composite points of compositional representations that induce weighted composition operators acting in the space of continuous functions defined in a metric compact set.

The space of continuous functions defined in a locally connected and connected compact of a nontrivial weighted composition operator acting on a uniform closed subpace the theorem expressing the necessary and sufficient condition for compactness has been proved. The theorem, which expresses the necessary and sufficient condition for compactness of a space of continuous functions defined in a locally connected compact of a nontrivial weighted composition operator acting on a uniform closed subspace, has been proven. The theorem on the compactness of weighted composition operators acting on uniform closed subspaces with the principle of compactness of the space of functions defined in a connected compact with a inside dense area everywhere is proved. A criterion was expressed for the nucleus of nontrivial compact weighted endomorphism of uniform algebra on a compact set containing a closed unit disk of the complex plane.The issues of closedness of the range of the non-trivial weighted composition operator and stability in the Hyer-Ulam sense were studied. Also, compactness criteria are given for weighted type composition operators, which are finite sums of weighted composition operators in the uniform space of the functions defined in the Hausdorff compact.

In general, the eigenvalues and corresponding eigenfunctions of the non-resonant endomorphism of the algebra of the convergent power series have been determined. In a special case, that is, the eigenvalues endomorfisms of the algebra of two-variable convergent power series are calculated and depending on the resonancing and non-resonancing states of these endomorphisms and the presence or absence of resonancing monoms, differential descriptions of the corresponding eigensubspaces are given with diffemorfism accuracy, and their dimensions are calculated.

Eigenvalues of weighted endomorphism induced by the Dencoy-Wolff fixed point in uniform algebras with analytical structure, eigenvalues of the endomorphism of the algebra of convergent power series with their corresponding subspaces, the bijective relation between the corresponding eigensubspaces was determined.

The main results of the dissertation work were published in the following works:

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