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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

STUDY OF DISTRIBUTIONS OF COMPLEX TOTAL PROCESSES AND THEIR BOUNDARY FUNCTIONALS

Specialty: 1208.01 Probability theory

Field of science: Mathematical Sciences

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GENERAL DESCRIPTION OF WORK

Relevance of the topic and the degree of development. This thesis is devoted to the study of processes associated with random walks, constructed from the sums of independent random variables and with a reflecting screen. Such processes are called semi-Markov walk processes with a reflecting screen.

On the one hand, the need for applications - reliability theory, queuing theory, inventory management theory, insurance theory, military affairs, etc. and on the other hand, the internal logic of the development of probability theory requires the study of more complex processes than jump-like Markov processes, for example, complex total processes of a semi-Markov walk with a reflecting screen at zero. Different models of research are proposed. The interest in these studies is due to their wide practical application. Intensive research of this model is underway, interesting results have been obtained, which are widely used in practice.

In this direction, important results were obtained by A.V. Skorokhod, B.P. Kharlamov, V.S. Korolyuk, A.F. Turbin, V.M. Shchurenkov, A.A. Borovkov, V.V.Gusak, T.I..Nasirova, T.A.Khaniev and others. The construction of mathematical models of real physical processes has brought to the fore the problems associated with the study of random processes generated by sequences of random variables

In these problems, physical processes are described by Markov processes, semi-Markov walks, and processes obtained from them using simple transformations, such as adding a constant drift, the presence of a delay screen, and summing up random processes. To a large extent, real processes studied using probabilistic methods are by their nature associated with the alternation of events of random duration.

However, in the case of a Markov process, the duration of the stay of the system in a certain state depends only on this state and has an necessarily exponential distribution. In the real world, the duration of the system's stay in a given state also depends on the state into which the process will go. The distribution of the duration of stay in the state can be arbitrary. Therefore, it became necessary to consider processes, although they are not Markovian, but have Markov moments, in which information about the past evolution of the process does not affect its future evolution.

Unlike Markov processes, semi-Markov processes can be in a fixed state for a very long time and the distribution of time in this position can have any law. And also this time interval depends not only on this position and also on the next position. Abroad, only A. Barovkov, V. Feller looked at absorbing screens in their robots. Here the system, as soon as it comes to the fixed screen, the process stops. T. Nasirova T. Khanyev, R. Alyev, T. Alyeva, Sh. Babaev, K. Omarova, E. Ibaev, U. Kerimova, B. Shamilova, etc. wrote on the topic of semi-Markov processes with a delay screen at zero in Azerbaijan a lot of scientific work.In this dissertation, reflective screen semi-Markov processes are studied. Suppose that the warehouse has a certain volume. At random times, a random amount of goods comes to the warehouse and the goods are taken out of the warehouse. There can be two cases. In the second case, if it is not possible to fulfill the request, to give the goods in the warehouse and say that there is nothing. In this case, we stipulate that the process should be constant. The probability of the first filling or emptying of the warehouse can be found with the help of complex aggregate processes - semi-marking process. This is the only way to ensure victory. Therefore, this fact alone shows that the process of reflecting semi-marks is very relevant for science.

Purpose and objectives of the study: Construction of complex summarized processes of semi-Markov walk with a reflecting screen. Write an integral equation for the Laplace-Stiltes transformation, the distribution function of semi-Markov processes with a reflecting screen at zero. Solve the integral equation for the Laplace-Stiltes transformation, the distribution function of semi-Markov processes with a reflecting screen at zero. Obtain a differential equation for the Laplace-Stiltes transform, the distribution function of semi-Markov processes with a reflecting screen at zero and solve it.

The purpose of the robots is to give a method for studying the distribution of complex summarized semi-Markov walk processes with a reflecting screen at zero and find their main boundary functionals, such as the Laplace transform in time and the Laplace-Stiltes transform in the phase of the conditional and unconditional joint distribution of the semi-Markov walk process of the moment of some level of the first intersection and jump through this level.

Find the generating function of the number of jumps at which the complex total process first reaches the level “a” (a> 0). Find the generating function of the number of jumps at which a complex process first reaches zero

Find in the strip [c, d] the sojourn distributions of the duration of time semi-Markov processes

Research methods: The dissertation applies the methods of probability theory and functional analysis

The main provisions for the defense: The dissertation obtained the following results

1. A complex summation process with a reflective screen is built.

2. Laplace-Stiltes transformation of the distribution function of the semi-Markov process with a reflective screen was found

3. The Laplace transform and the Laplace-Stiltyes transformation function were found when the reflective screen semi-Markov rotation process reached the level a for the first time and the joint length distribution function of the jump length over this level.

4. The function generating the number of leaps for which the complex sum process first reached the level "a" (a> 0) has been found.

5. The function of generating the number of jumps where the complex sum process reaches the first zero is found. The average values of the process for this case are found.

6. In the part [c, d] the integral equation of the distribution function with delayed argument for the remaining time of the semi-markov process is constructed.

Scientific research: The process of semi-markov wandering with a reflecting screen is a well-known scientific novelty

Theoretical and practical value of the research: The results obtained in the dissertation work are both theoretical and practical. These results can be applied in science, economics and military affairs.

Approbation and application: The results of the dissertation were repeatedly reported at the seminars of the "Probable methods of management" department of the Institute of Information and Management of ANAS, at the seminars of the "Functional analysis" department of the Institute of Mathematics and Mechanics of ANAS. The main results of the dissertation work on "Actual problems of mathematics and mechanics" dedicated to the 90th anniversary of national leader Heydar Aliyev (Baku, 2013), "Modern problems of mathematics and mechanics" dedicated to the 80th anniversary of Academician A. Hajiyev (Baku, 2017) Presented at the International Conference of Scientists, "Information Systems and Technologies: Achievements and Prospects" (Sumgayit, 2018)

Name of the where the dissertation work was performed:

Institute of Mathematics and Mechanics of ANAS

The total volume of the dissertation in the signs with the indication of the volume of each structural section in the section:

On the topic of the dissertation, 9 works were published, the list of which is given at the end of the summary. The dissertation consists of 42,000 characters, three chapters: the first - 32,000 characters, the second - 30,000 characters, the third - 54,000 characters; result-2000 characters, bibliography. The main text consists of 160,000 characters.

THE CONTENT OF THE WORK

In the first chapter of this thesis, a jump-like distribution with a reflecting screen at zero is investigated.

Let a sequence of random variables be given on a probability space $(\Omega, F, P(\cdot))$, where independent, equally distributed, positive random variables $\left\{\xi_k^+, \eta_k^+, \xi_k^-, \eta_k^-\right\}_{k=\overline{1,\infty}}$. As X (t), in a special case we can take the amount of product in stock up to the moment t.

If we denote X (t) as a compound sum, we must accept the following conditions as a requirement of the Reflective Screen problem.

We denote

$$X(t) = \left|S_{k-1} + \ldots + \eta^+_{\nu(\tau_{k-1})} + \eta^+_{\nu(\tau_k)} - \eta^-_k\right|,$$

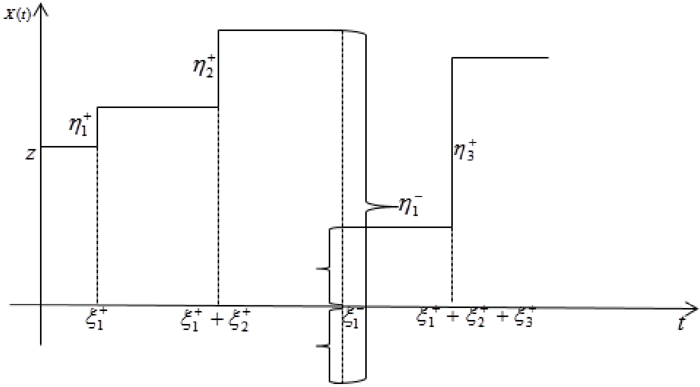
$$\tau_k^{\pm} = \sum_{i=1}^{k} \xi_i^{\pm}; k = 1,2,..\infty;\ \tau_0^{\pm} = 0,$$

$$S_0 = z,$$

where

$$v^{\pm}(t) = \min\left\{k : \sum_{i=1}^{k+1} \xi_i^{\pm} > t\right\}.$$

If we call the process a complex semi-Markov walk process with a reflecting screen at zero. One of the realizations of the process X (t) has the form:



$v^{\pm}(t)$ number of positive or negative jumps in time t

Our goal is to find an explicit form of the Laplace-Stieltjes transformation in the phase of the unconditional distribution of the process

We denote

$$R(t,x|z) = P\{X(t) < x | X(0) = z\}-$$

Process distribution function

$$\tilde{R}(\theta,x|z) = \int_{t=0}^{\infty} e^{-\theta t} R(t,x|z)dt, \theta > 0 \quad -$$

Laplace transform of the distribution function of the process

$$\tilde{\tilde{R}}(\theta,\alpha|z) = \int_{x=0}^{\infty} e^{-\alpha x} d_x \, \tilde{R}(\theta,x|z), \; \alpha > 0 -$$

Laplace-Stiltyes transformation of the distribution function

Drawing up an integral equation for $\tilde{\tilde{R}}(\theta,\alpha \mid z)$.

By the formula of total probability, we have

$$P\{X(t)<x|X(0)=z\}=P\{X(t)<x,\xi_1^->t|X(0)=z\}+$$
$$+P\{X(t)<x,\xi_1^-<t|X(0)=z\}$$
$$P\{X(t)<x|X(0)=z\}=P\{X^+(t)<x,\xi_1^->t|X(0)=z\}+$$
$$+\int_{s=0}^{t}\int_{y=0}^{\infty}P\{\xi_1^-\in ds,X(s)\in dy|X(0)=z\}P\{X(t-s)<x|X(0)=y\}$$

Then we can write such a formula for the distribution function:

$$R(t,x|z)=P\{X^+(t)<x,\xi_1^->t|X(0)=z\}+\int_{s=0}^{t}\int_{y=0}^{\infty}P\{\xi_1^-\in ds,X(s)\in dy|R(t-s,x|y)$$

Theorem 1

The Laplace-Stiltes transform of the distribution of a semi-Markov walk with a reflecting screen at zero satisfies the following integral equation:

$$\tilde{\tilde{R}}(\theta,\alpha|z)=e^{-\alpha z}\int_{t=0}^{\infty}e^{-\theta t}P\{\xi_1^->t\}dt$$
$$+\sum_{k=1}^{\infty}\int_{x=z}^{\infty}e^{-\alpha x}d_xP\left\{\sum_{i=1}^{k}\eta_1^+<x-z\right\}\int_{t=0}^{\infty}e^{-\theta t}P\{\nu^+(t)=k\}P\{\xi_1^->t\}+$$
$$+\int_{y=o}^{\infty}\tilde{\tilde{R}}(\theta,\alpha|y)d_yP\{\eta_1^-<+y+z\}\int_{t=0}^{\infty}e^{-\theta t_-}P\{\xi_1^+>t\}dP\{\xi_1^-<t\}-$$
$$-\int_{y=o}^{z}\tilde{\tilde{R}}(\theta,\alpha|y)d_yP\{\eta_1^-<-y+z\}\int_{t=0}^{\infty}e^{-\theta t_-}P\{\nu^+(t)=k\}dP\{\xi_1^-<t\}+$$
$$+\int_{y=o}^{\infty}\tilde{\tilde{R}}(\theta,\alpha|y)\sum_{k=1}^{\infty}\int_{\gamma=0}^{\infty}d_yP\{\eta_1^-<z+\gamma+y\}d_yP\left\{\sum_{i=1}^{k}\eta_i^+<\gamma\right\}\int_{t=0}^{\infty}e^{-\theta t_-}P\{\nu^+(t)=k\}dP\{\xi_1^-<t\}+$$
$$+\int_{y=o}^{\infty}\tilde{\tilde{R}}(\theta,\alpha|y)\sum_{k=1}^{\infty}\int_{\gamma=\max(0,y=z)}^{\infty}d_yP\{\eta_1^-<z+\gamma-y\}\times$$
$$\times d_yP\left\{\sum_{i=1}^{k}\eta_i^+<\gamma\right\}\int_{t=0}^{\infty}e^{-\theta t}P\{\nu^+(t)=k\}dP\{\xi_1^-<t\}$$

Let us solve this integral equation under the following conditions. Suppose that the quantities and time of a product are subject to exponential law

$$P\{\xi_1^{\pm} < t\} = \begin{cases} 0, & t < 0, \\ 1 - e^{-\lambda_{\pm} t}, & t > 0, \quad \lambda_{\pm} > 0, \end{cases}$$

$$P\{\eta_1^{\pm} < t\} = \begin{cases} 0, & t < 0 \\ 1 - e^{-\eta_{\pm} t}, & t > 0, \eta_{\pm} > 0, \end{cases}$$

Step numbers occur by Poisson's law.

$$P\{\nu^{+}(t) = k\} = \frac{(\lambda_{+} t)^k}{k!} e^{-\lambda_{+} t},$$

$$d_{\gamma} P\left\{\sum_{i=1}^{k} \eta_i^{+} < \gamma\right\} = \frac{\mu_{+}^{k} \gamma^{k-1}}{(k-1)!} e^{-\mu_{+}\gamma}$$

We could take random quantities as a larger order but for simplicity's sake. Thus, the integral equation of the Laplace-Stiltyes transformation is reduced to an ordinary differential equation.

$$\frac{d^2 \tilde{\tilde{R}}(\theta,\alpha|z)}{dz^2} + \left(-\frac{(\lambda_{-} + \theta)\mu_{+}}{\lambda_{+} + \lambda_{-} + \theta} + \mu_{-} + \frac{\lambda_{-}\mu_{-}}{\lambda_{+} + \lambda_{-} + \theta} \right) \frac{d\tilde{\tilde{R}}(\theta,\alpha|z)}{dz} -$$

$$-\frac{(2\lambda_{-} + \theta)\mu_{+}\mu_{-}}{\lambda_{+} + \lambda_{-} + \theta} \tilde{\tilde{R}}(\theta,\alpha|z) = \frac{(\alpha - \mu_{-})(\alpha + \mu_{+}) e^{-\alpha z}}{\lambda_{+} + \lambda_{-} + \theta}$$

And it is solved:

$$\tilde{\tilde{R}}(\theta,\alpha|z) = C_1(\theta) e^{k_1} + C_2(\theta) e^{k_2} + C e^{-\alpha z},$$

$$C = \frac{(\alpha - \mu_{-})(\alpha + \mu_{+})}{(\lambda_{+} + \lambda_{-} + \theta)(\alpha + k_1)(\alpha + k_2)}$$ taken in the form of

Chapter 2 is devoted to the Laplace-Stiltes transform of the joint distribution of the moment of the first crossing of some level “a” (a> 0) and jumping over this level by a complex process of semi-Markov walk with a reflecting screen at zero.

The purpose of this chapter is to find an explicit form of the Laplace-Stiltes transform of the joint distribution. $K(t,\gamma \mid X(0)=z)=P\{\tau_a<t,\gamma>a \mid X(0)=z\}$

In addition to the previous data, it is given here in addition and the quantities indicate the moment of passing level a and the length of time it exceeds this level, respectively.

Theorem 2

The distribution function of the process is as follows

$$\tilde{\tilde{K}}(\theta,\chi \mid z)=-\int_{t=0}^{\infty} e^{-\theta t}P\{\xi_1^->t\}P\{\nu^+(t)=0\}dt\int_{\gamma=0}^{\infty} d_\gamma \varepsilon(a+\gamma-z)-$$

$$-\int_{t=0}^{\infty} e^{-\theta t}P\{\xi_1^->t\}\sum_{k=1}^{\infty}P\{\nu^+(t)=k\}\int_{\gamma=0}^{\infty} e^{-\chi\gamma}d_\gamma P\{\sum_{i=1}^{k}\zeta_1^+<a+\gamma-z\}-$$

$$-\int_{y=0}^{z}\tilde{\tilde{K}}(\theta,\gamma \mid y)d_y P\{\zeta_1^-<-y+z\}\int_{t=0}^{\infty}e^{-\theta t}P\{\nu^+(t)=0\}d_t P\{\xi_1^-<t\}-$$

$$-\int_{y=o}^{z}\tilde{\tilde{K}}(\theta,\gamma \mid y)\int_{h=0}^{a-z}d_y P\{\zeta_1^-<-y+h+z\}d_h\sum_{k=1}^{\infty}P\{\sum_{i=1}^{k}\zeta_i^+<h\}\int_{t=0}^{\infty}e^{-\theta t}P\{\nu^+(t)=k\}\times$$

$$\times d_t P\{\xi_1^-<t\}-\int_{y=z}^{a}\tilde{\tilde{K}}(\theta,\gamma \mid y)d_y P\{\zeta_1^-<-y+z\}\int_{t=0}^{\infty}e^{-\theta t}P\{\nu^+(t)=0\}d_t P\{\xi_1^-<t\}-$$

$$-\int_{y=z}^{a}\tilde{\tilde{K}}(\theta,\gamma \mid y)\int_{h=y-z}^{a-z}d_y P\{\zeta_1^-<-y+h+z\}d_h\sum_{k=1}^{\infty}P\{\sum_{i=1}^{k}\zeta_i^+<h\}\int_{t=0}^{\infty}e^{-\theta t}P\{\nu^+(t)=k\}\times$$

$$\times d_t P\{\xi_1^-<t\}++\int_{y=0}^{a}\tilde{\tilde{K}}(\theta,\gamma \mid y)d_y P\{\zeta_1^-<y+z\}\int_{t=0}^{\infty}e^{-\theta t}P\{\nu^+(t)=0\}d_t P\{\xi_1^-<t\}+$$

$$+\int_{y=0}^{a}\tilde{\tilde{K}}(\theta,\gamma \mid y)\int_{h=0}^{a-z}d_y P\{\zeta_1^-<y+h+z\}\times$$

$$\times d_h\sum_{k=1}^{\infty}P\{\sum_{i=1}^{k}\zeta_i^+<h\}\int_{t=0}^{\infty}e^{-\theta t}P\{\nu^+(t)=k\}d_t P\{\xi_1^-<t\}$$

Let us solve the integral equation in the particular case when

$$P\{\xi_1^\pm<t\}=\begin{cases}0, t<0\\ 1-e^{-\lambda_\pm t}, \lambda_\pm>0, t>0\end{cases}$$

$$P\{\zeta_1^{\pm} < x\} = \begin{cases} 0, x < 0 \\ 1 - e^{-\mu_{\pm} x}, x > 0, \mu_{\pm} > 0 \end{cases}$$

As a result, we obtain an inhomogeneous differential equation of the second order with constant coefficients

$$\tilde{\tilde{K}}''(\theta,\chi,z) + (\mu_- + \frac{\mu_+\lambda_+}{\lambda_+ + \lambda_- + \theta})\tilde{\tilde{K}}'(\theta,\chi,z) +$$

$$+\left[\frac{\lambda_+\mu_+\mu_-}{\lambda_+ + \lambda_- + \theta} + \frac{\lambda_+\lambda_-\mu_+\mu_-}{(\lambda_+ + \lambda_- + \theta)^2}\right]\tilde{\tilde{K}}(\theta,\chi,z) =$$

$$= \frac{(\mu_- - \chi)(\mu_+(\lambda_- + \theta) + \chi(\lambda_+ + \lambda_- + \theta))}{(\lambda_+ + \lambda_- + \theta)^2} e^{(a-z)\chi}$$

As a result, we obtain an inhomogeneous differential equation of the second order with constant coefficients

$$k_{1;2}(\theta) =$$

$$= \frac{-(\mu_- + \frac{\mu_+\lambda_+}{\lambda_+ + \lambda_- + \theta}) \pm \sqrt{(\mu_- + \frac{\mu_+\lambda_+}{\lambda_+ + \lambda_- + \theta})^2 - 4\left[\frac{\lambda_+\mu_+\mu_-}{\lambda_+ + \lambda_- + \theta} + \frac{\lambda_+\lambda_-\mu_+\mu_-}{(\lambda_+ + \lambda_- + \theta)^2}\right]}}{2}$$

and obtained a solution to the differential equation

$$\tilde{\tilde{K}}(\theta,\chi,z) = \frac{(\mu_- - \chi)((\mu_+(\lambda_- + \theta) + \chi(\lambda_+ + \lambda_- + \theta))}{(\lambda_+ + \lambda_- + \theta)^2(\chi + k_1(\theta))(\chi + k_2(\theta))} e^{\chi(a-z)} +$$

$$+C_1(\theta)e^{k_1(\theta)z} + C_2(\theta)e^{k_2(\theta)z}$$

where $C_1(\theta)$ and $C_2(\theta)$ constant relative z

Chapter 3 is devoted to the study of the function of the number of jumps, during which the complex process of semi-Markov wandering first reaches the level "a" (a> 0)

Let on the probability space $(\Omega, \mathcal{F}, P(\cdot))$ given a sequence of mutually independent identically distributed positive random variables

$$\xi_k^+ \ , \eta_k^+, \xi_k^-, \eta_k^-, \ k = \overline{1,\infty}$$

We introduce the following notation

$\nu^{\pm}(t) = \min\left\{k : \sum_{i=1}^{k+1} \xi_i^{\pm} > t\right\}$ - the number of positive jumps of the process

$X^{\pm}(t)$ in time (t)

$$X^{\pm}(t) = \sum_{i=1}^{\nu^{\pm}(t)} \eta_i^{\pm}$$

$$X(t) = X^{+}(t) - X^{-}(t)$$

The process X(t) will be called a complex semi-Markov walk process.

The goal is to find an explicit form of the generating function of the number of jumps at which the process first reaches the level ``a"

Let be

$$X(0) = z > 0$$

Let us denote ν_1^a the number of jumps of the process at which it first reaches the level `` a "

The generating function is as follows

$$\Psi(u \mid z) = \sum_{k=1}^{\infty} u^k P\left\{\nu_1^a = k \mid X(0) = z\right\}, \ |u| \le 1.$$

Theorem 3: $\Psi(u \mid z)$ satisfies the following integral equation

$$\Psi(u \mid z) = uP\left\{\eta_1^+ > a - z\right\} + u \int_{y=z}^{a} \Psi(u \mid y) dy P\left\{\eta_1^+ < y - z\right\} P\left\{\xi_1^+ < \xi_1^-\right\} +$$

$$+ u \int_{y=z}^{a} \Psi(u \mid y) \int_{x=y}^{\infty} dy \sum_{m=1}^{\infty} P\left\{\eta_1^- + \ldots + \eta_m^- < x - y\right\} \times$$

$$\times \int_{t=0}^{\infty} P\left\{\nu^-(t) = m\right\} d_t P\left\{\xi_1^+ - \xi_1^- < t\right\} d_y P\left\{\eta_1^+ < y\right\} +$$

$$+u\int_{y=-\infty}^{z}\Psi(u\mid y)\int_{x=z}^{\infty}dy\sum_{m=1}^{\infty}\mathrm{P}\left\{\eta_1^- + \ldots + \eta_m^- < x-y\right\}\times$$

$$\times\int_{t=0}^{\infty}\mathrm{P}\left\{\nu^-(t)=m\right\}d_t\mathrm{P}\left\{\xi_1^+ - \xi_1^- < t\right\}d_y\mathrm{P}\left\{\eta_1^+ < y\right\}$$

If we solve this equation under the above conditions, that is, if we write the following in the integral equation and solve it in a special case:

$$P\{\xi_1^{\pm} < t\} = \begin{cases} 0, t < 0 \\ 1 - e^{-\lambda_{\pm} t} \end{cases} \quad t > 0,\ \lambda_+ > 0, \quad \lambda > 0$$

$$P\{\eta_1^{\pm} < x\} = \begin{cases} 0, x < 0 \\ 1 - e^{-\mu_{\pm} t} \end{cases} \quad x > 0,\ \mu_+ > 0,\ \mu > 0$$

$$\mathrm{P}\left\{\xi_1^+ < \xi_1^-\right\} = \frac{\lambda_+}{\lambda_+ + \lambda_-},$$

$$\mathrm{P}\left\{\xi_1^- < \xi_1^+\right\} = \frac{\lambda_-}{\lambda_+ + \lambda_-},$$

$$d_t\mathrm{P}\left\{\xi_1^+ - \xi_1^- < t\right\} = \frac{\lambda_+\lambda_-}{\lambda_+ + \lambda_-}e^{-\lambda_+ t}dt,$$

$$P\left\{\nu^{\pm}(t) = m\right\} = \frac{(\lambda_{\pm}t)^m}{m!}e^{-\lambda_{\pm}t}$$

$$d_yP\left\{\eta_1^- + \eta_2^- + \ldots + \eta_m^- < y\right\} = \mu_-\frac{(\mu_- y)^{m-1}}{(m-1)!}e^{-\mu_- y}dy$$

We obtain a second-order differential equation

$$\Psi''(u\,|\,z)+[-\mu_+ +\frac{\lambda_+\mu_-}{\lambda_+ +\lambda_-}+\frac{\lambda_+\mu_+}{\lambda_+ +\lambda_-}u]\Psi'(u\,|\,z)+[-\frac{\lambda_+\mu_+\mu_-}{\lambda_+ +\lambda_-}+$$

$$\frac{\lambda_+^2\mu_+\mu_-}{(\lambda_+ +\lambda_-)^2}u+\frac{\lambda_+\lambda_-^2\mu_+\mu_-}{(\lambda_+ +\lambda_-)^3}u]\Psi(u\,|\,z)=0$$

Its solution is shown below

$$\Psi(u\,|\,z)=\frac{(\lambda_+ +\lambda_-)(k_1(u)-\mu_+)e^{k_1(u)z}}{\lambda_+\mu_+e^{k_1(u)a}-\frac{\lambda_-^2\mu_-(\lambda_-\mu_+ +\lambda_+\mu_-)(k_1(u)+\mu_+)}{[k_1(u)(\lambda_+ +\lambda_-)+\lambda_+\mu_-](\mu_+(\lambda_+ +\lambda_-)+\lambda_+\mu_-)}e^{\mu_+a}}$$

This chapter also finds the function of the number of steps taken before the semi-Markov process, which reflects zero, reaches zero.

Let us denote v_0 the number of steps taken before the X (t) process reaches zero for the first time.

The generating function will be as follows.

$$\Psi_0(u\,|\,z)=\sum_{k=1}^{\infty}u^kP\{v_0=k\,|\,X\,|(0)=z\}.$$

Theorem 4. The integral equation for the generating function is:

$$\Psi_0(u\,|\,z)=u\int_{h=0}^{\infty}P\{\eta_1^->z+h\}d_h\int_{t=0}^{\infty}\sum_{m=0}^{\infty}P\left\{\sum_{i=1}^{m}\eta_i^+<h\right\}P\{v^+(t)=m\}d_tP(\xi_1^-<t)-$$

$$-u\int_{y=0}^{z}\Psi_0(u\,|\,y)d_y\int_{h=0}^{\infty}P\{\eta_1^-<z+h-y\}d_h\int_{t=0}^{\infty}\sum_{m=0}^{\infty}P\left\{\sum_{i=1}^{m}\eta_i^+<h\right\}P\{v^+(t)=m\}d_tP(\xi_1^-<t)-$$

$$-u\int_{y=0}^{z}\Psi_0(u\,|\,y)d_y\int_{h=y-z}^{\infty}P\{\eta_1^-<z+h-y\}d_h\int_{t=0}^{\infty}\sum_{m=0}^{\infty}P\left\{\sum_{i=1}^{m}\eta_i^+<h\right\}P\{v^+(t)=m\}d_tP(\xi_1^-<t).$$

Let's solve this equation under special conditions

$$P\{\xi_1^{\pm}<t\}=\begin{cases}0, & t<0,\\ 1-e^{-\mu_{\pm}t}, & t>0,\lambda_{\pm}>0,\end{cases}$$

$$P\{\eta_1^{\pm} < t\} = \begin{cases} 0, & t < 0, \\ 1 - e^{-\mu_{\pm} t}, & t > 0, \mu_{\pm} > 0. \end{cases}$$

We obtain the differential equation below:

$$\Psi''(u \mid z) + \left(\frac{\lambda_+ \lambda_-}{\lambda_+ + \lambda_-} - \mu_+ + \mu_- \right) \Psi'(u \mid z) +$$

$$+ \left(\frac{\lambda_+ \mu_+ \mu_-}{\lambda_+ + \lambda_-} - \mu_+ + \mu_- - \frac{\lambda_+ \mu_+ u}{(\lambda_+ + \lambda_-)^2} \right) \Psi(u \mid z) = 0$$

Thus a simple formula is obtained for the generating function

$$\Psi_0(u \mid z) = C_1(u) e^{k_1(u) z}$$

$$C_1(u) =$$

$$= \frac{\dfrac{\lambda_- u}{\lambda_+ \lambda_-} + \dfrac{\lambda_+ \lambda_- \mu_+ u}{(\lambda_+ + \lambda_-)^2 (\mu_+ + \mu_-) + \lambda_+ \mu_+ (\lambda_+ + \lambda_-)}}{1 - \dfrac{\lambda_+ \mu_- u}{(\lambda_+ + \lambda_-)[\mu_- k_1(u)]} - \dfrac{\lambda_+ \lambda_- \mu_+ u}{(\lambda_+ + \lambda_-)(\mu_+ + \mu_-) - \lambda_+ \mu_+} \dfrac{1}{\lambda_- \mu_+ - (\lambda_+ + \lambda_-) k_1(u)}}.$$

The mathematical expectation and variance were found for the number of jumps in a complex semi-Markov walk process, in which the process first reaches zero

$$K^2(u) + [-\mu_+ + \frac{\lambda_+ \mu_-}{\lambda_+ + \lambda_-} + \frac{\lambda_+ \mu_+}{\lambda_+ + \lambda_-} u] K(u) +$$

$$+ \left[-\frac{\lambda_+ \mu_+ \mu_-}{\lambda_+ + \lambda_-} + \frac{\lambda_+^2 \mu_+ \mu_-}{(\lambda_+ + \lambda_-)^2} u + \frac{\lambda_+ \lambda_-^2 \mu_+ \mu_-}{(\lambda_+ + \lambda_-)^3} u \right] = 0.$$

Then we represent the roots of the characteristic equation in the following form

$$K_{1,2}(u) = \frac{\left(\dfrac{\lambda_+ \lambda_-}{\lambda_+ \lambda_-} - \mu_+ + \mu_- \right)}{2} \pm$$

$$\pm\frac{\sqrt{\left(\frac{\lambda_+\lambda_-}{\lambda_+\lambda_-}-\mu_++\mu_-\right)^2-4\left(\frac{\lambda_+\lambda_-}{\lambda_+\lambda_-}-\mu_++\mu_--\frac{\lambda_+\lambda_-\mu_+u}{(\lambda_++\lambda_-)^2}\right)}}{2}$$

We denote

$$A=-\left(\frac{\lambda_+\lambda_-}{\lambda_+\lambda_-}-\mu_++\mu_-\right)$$

$$B=-\left(\frac{\lambda_+\lambda_-}{\lambda_+\lambda_-}-\mu_++\mu_-\right)^2-4\left(\frac{\lambda_+\lambda_-}{\lambda_+\lambda_-}-\mu_+\mu_-\right)$$

$$D=4\left(\frac{\lambda_+\lambda_-}{(\lambda_+\lambda_-)^2}\right)$$

Then the roots of the characteristic equation will be of the following form

$$K(u)=A\pm\frac{\sqrt{B+Du}}{2}\qquad K(1)=A\pm\frac{\sqrt{B+Du}}{2}$$

$$K'(u)=\pm\frac{D}{4\sqrt{B+Du}}\qquad K'(1)=\pm\frac{D}{4\sqrt{B+D}}$$

$$\Psi_0(u\mid z)=C_1(u)e^{k_1(u)z}$$

Differentiating with respect to u, we obtain

$$\Psi_0'(u\mid z)=\left(C_1'(u)+C_1(u)K_1'(u)z\right)e^{k_1(u)z}$$

Here

$$C_1(u)=$$

$$\frac{\frac{\lambda_-u}{\lambda_++\lambda_-}+\frac{\lambda_+\lambda_-\mu_+u}{(\lambda_++\lambda_-)^2(\mu_++\mu_-)-\lambda_+\mu_+(\lambda_++\lambda_-)}}{1-\frac{\lambda_-\mu_-u}{(\lambda_++\lambda_-)\left[\mu_--k_1(u)\right]}-\frac{\lambda_+\lambda_-\mu_+u}{(\lambda_++\lambda_-)(\mu_++\mu_-)-\lambda_+\mu_+}\frac{1}{\mu_+(\lambda_++\lambda_-)-\lambda_+\mu_+-(\lambda_++\lambda_-)k_1(u)}}$$

$$C_1(u)=$$

$$\frac{\dfrac{\lambda_-}{\lambda_+ + \lambda_-} + \dfrac{\lambda_+\lambda_-\mu_+}{(\lambda_+ + \lambda_-)^2(\mu_+ + \mu_-) - \lambda_+\mu_+(\lambda_+ + \lambda_-)}}{\dfrac{1}{u} - \dfrac{\lambda_-\mu_-}{(\lambda_+ + \lambda_-)[\mu_- - k_1(u)]} - \dfrac{\lambda_+\lambda_-\mu_+}{(\lambda_+ + \lambda_-)(\mu_+ + \mu_-) - \lambda_+\mu_+} \dfrac{1}{\mu_+(\lambda_+ + \lambda_-) - \lambda_+\mu_+ - (\lambda_+ + \lambda_-)k_1(u)}}$$

We denote

$$\Phi = \frac{\lambda_-}{\lambda_+ + \lambda_-} + \frac{\lambda_+\lambda_-\mu_+}{(\lambda_+ + \lambda_-)^2(\mu_+ + \mu_-) - \lambda_+\mu_+(\lambda_+ + \lambda_-)}$$

$$H = \frac{\lambda_-\mu_-}{\lambda_+ + \lambda_-}$$

$$E = \frac{\lambda_+\lambda_-\mu_+}{(\lambda_+ + \lambda_-)((\lambda_+ + \lambda_-)(\mu_+ + \mu_-) - \lambda_+\mu_+)}$$

$$Y = \frac{\mu_+(\lambda_+ + \lambda_-) - \lambda_+\mu_+}{\lambda_+ + \lambda_-}$$

Then

$$C_1(u) = \frac{\Phi}{\dfrac{1}{u} - \dfrac{H}{[\mu_- - K_1(u)]} - \dfrac{E}{У - K_1(u)}}$$

Differentiating with respect to u, we obtain

$$C_1'(u) =$$

$$= \frac{\Phi}{\left(\dfrac{1}{u} - \dfrac{H}{[\mu_- - K_1(u)]} - \dfrac{E}{У - K_1(u)}\right)^2} \left(\frac{1}{u^2} + \frac{H}{[\mu_- - K_1(u)]^2} K_1'(u) + \frac{E}{[У - K_1(u)]^2} K_1'(u)\right)$$

$$C_1(u) = \frac{\Phi}{\dfrac{1}{u} - \dfrac{H}{[\mu_- - K_1(u)]} - \dfrac{E}{У - K_1(u)}}$$

If $u = 1$, we obtain

$$C_1(1)=\frac{\Phi}{1-\frac{H}{[\mu_- - K_1(1)]}-\frac{E}{У-K_1(1)}}$$

$$C_1'(u)=\frac{\Phi}{\left(\frac{1}{u}-\frac{H}{[\mu_- - K_1(u)]}-\frac{E}{У-K_1(u)}\right)^2}\left(\frac{1}{u^2}+\frac{H}{[\mu_- - K_1(u)]^2}K_1'(u)+\frac{E}{[У-K_1(u)]^2}K_1'(u)\right)$$

If $u=1$,we obtein

$$C_1'(1)=\frac{\Phi}{\left(1-\frac{H}{[\mu_- - K_1(1)]}-\frac{E}{У-K_1(1)}\right)^2}\left(1+\frac{H}{[\mu_- - K_1(1)]^2}K_1'(1)+\frac{E}{[У-K_1(1)]^2}K_1'(1)\right)$$

As a result, we get the mathematical expectation of the number of jumps at which the complex process of semi-Markov walk reaches zero for the first time

$$M(\nu_1^0)=\Psi_0'(1\,|\,z)=\left(C_1'(1)+C_1(1)K_1'(1)z\right)e^{K_1(1)z}=$$

$$\left(\frac{\Phi}{\left(1-\frac{H}{[\mu_- - K_1(1)]}-\frac{E}{У-K_1(1)}\right)^2}\left(1+\frac{H}{[\mu_- - K_1(1)]^2}K_1'(1)+\frac{E}{[У-K_1(1)]^2}K_1'(1)\right)+\right.$$

$$\left.+\frac{\Phi}{1-\frac{H}{[\mu_- - K_1(1)]}-\frac{E}{У-K_1(1)}}K_1'(1)z\right)e^{k_1(1)z}$$

We can find the variance

$$D(\nu_1^0)=\Psi_0''(1\,|\,z)+\Psi'(1\,|\,z)-(\Psi'(1\,|\,z))^2$$

here

$$\Psi_0''(1\,|\,z)=\{\left(C_1'(1)+C_1(1)K_1'(1)z\right)'+K_1{}'(1)z\left(C_1'(1)+C_1(1)K_1'(1)z\right)\}e^{k_1(1)z}$$

And we get the variance

$$D(\nu_1^0)=[\left(C_1'(1)+C_1(1)K_1'(1)z\right)'+K_1{}'(1)z\left(C_1'(1)+C_1(1)K_1'(1)z\right)]e^{K_1(1)z}+$$

$$+\left(C_1'(1)+C_1(1)K_1'(1)z\right)e^{K_1(1)z}-[\left(C_1'(1)+C_1(1)K_1'(1)z\right)e^{K_1(1)z}]^2$$

This chapter also investigates an integral equation with a lagging argument in semi-Markov walk processes

In this paper, we find the Laplace transform of the distribution of the sojourn time of the difference process of a semi-Markov walk.

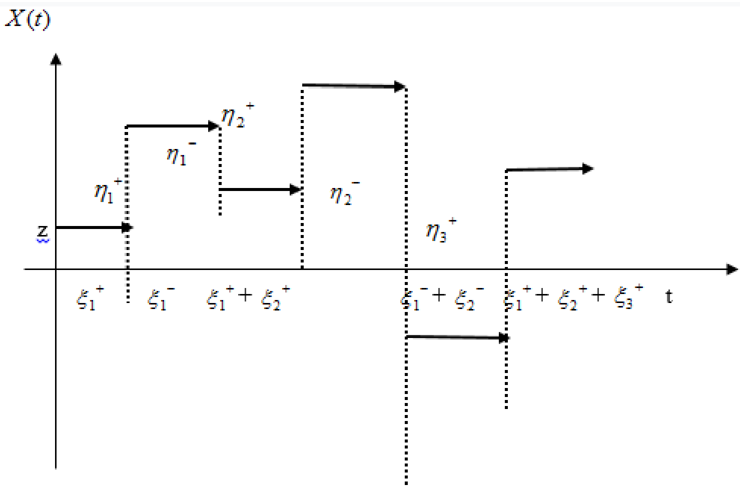
Let two independent lonely distributed random variables of the sequence

$\{\xi_k^+, \eta_k^+\}_{k=\overline{1,\infty}}$ и $\{\xi_k^-, \eta_k^-\}_{k=\overline{1,\infty}}$, pairs in each sequence independent and $\xi_k^\pm > 0,\ \eta_k^\pm > 0$. Using these random variables, we construct the following semi-Markov walk processes

$$X^+(t) = \sum_{i=1}^{k-1} \eta_i^+, \quad \text{if} \quad \sum_{i=1}^{k-1} \xi_i^+ \le t < \sum_{i=1}^{k} \xi_i^+ ,$$

$$X^-(t) = \sum_{i=1}^{k-1} \eta_i^- , \quad \text{if} \quad \sum_{i=1}^{k-1} \xi_i^- \le t < \sum_{i=1}^{k} \xi_i^- .$$

Process $X(t) = X^+(t) - X^-(t)$ will be called a difference process of a semi-Markov walk. One of its implementations will be as follows



The goal is to find the Laplace transform of the distribution of the duration of the residence time of the process X(t)

We assume that the random variable is distributed exponentially. Then it becomes a complex difference Markov process

Let the process X(t) run in a strip [c , d] , c>0, d>0.

We denote

$$A_z = \left\{ \inf_{0\le s\le t} X(s) > c; \sup_{0\le s\le t} X(s) < d \mid X(0) = z \right\}, \quad z > 0,$$

and

$$K(t; c, d \mid X(0) = z) = P\left\{ \inf_{0\le s\le t} X(s) > c; \sup_{0\le s\le t} X(s) < d \mid X(0) = z \right\}. \quad (0.13)$$

Let's write the Laplace transform

$$\tilde{K}(\theta; c, d \mid X(0) = z) = \int_{t=0}^{\infty} e^{-\theta t} P\left\{ \inf_{0\le s\le t} X(s) > c; \sup_{0\le s\le t} X(s) < d \mid X(0) = z \right\} dt$$

Theorem 5: Then the integral equation for the Laplace transform of the distribution of the sojourn time of the difference process of a semi-Markov walk will be as follows

$$\begin{aligned}
.\tilde{K}(\theta / z) = {} & \frac{1}{\lambda_+ + \lambda_- + \theta} - \frac{\lambda_+\lambda_-\mu_-}{(\lambda_+ + \theta)\theta} e^{-\mu_- d} \int_{y=c}^{d} \tilde{K}(\theta \mid y) e^{\mu_- y} dy \\
& + \frac{\lambda_+\lambda_-\mu_-}{(\lambda_+ + \theta)(\lambda_- + \theta)} e^{-\frac{\lambda_+\mu_+ + (\mu_+ + \mu_-)\theta}{\lambda_- + \theta} d} e^{\frac{\mu_+\theta}{\lambda_+ + \theta} z} \int_{y=c}^{d} \tilde{K}(\theta \mid y) e^{\mu_- y} dy \\
& + \frac{\lambda_+\lambda_-\mu_+\mu_-}{(\lambda_+ + \theta)(\lambda_- + \theta)} e^{\frac{\mu_+\theta}{\lambda_+ + \theta} z} \int_{y=z}^{d} \tilde{K}(\theta \mid y) e^{-\frac{\mu_+\theta}{\lambda_+ + \theta} y} \int_{x=0}^{d-y} e^{-\frac{[(\lambda_+ + \theta)\mu_- + \mu_+\theta]}{\lambda_+ + \theta} x} dx dy \\
& - \frac{\lambda_+\lambda_-\mu_+\mu_-}{(\lambda_+ + \theta)(\lambda_- + \theta)} e^{\frac{\mu_+\theta}{\lambda_+ + \theta} z} \int_{y=c}^{z} \tilde{K}(\theta \mid y) e^{-\frac{\mu_+\theta}{\lambda_+ + \theta} y} \int_{x=z-y}^{d-y} e^{-\frac{[(\lambda_+ + \theta)\mu_- + \mu_+\theta]}{\lambda_+ + \theta} x} dx dy \\
& - \frac{\lambda_+\lambda_-\mu_+}{(\lambda_- + \theta)\theta} e^{\mu_+ b} \int_{y=c}^{d} \tilde{K}(\theta \mid y) e^{-\mu_+ y} dy \\
& + \frac{\lambda_+\lambda_-\mu_+}{(\lambda_+ + \theta)(\lambda_- + \theta)} e^{\frac{\lambda_-\mu_+ + (\mu_+ + \mu_-)\theta}{\lambda_- + \theta} b} e^{\frac{\mu_-\theta}{\lambda_- + \theta} z} \int_{y=c}^{d} \tilde{K}(\theta \mid y) e^{-\mu_+ y} dy
\end{aligned}$$

$$+\frac{\lambda_+\lambda_-\mu_+}{(\lambda_++\theta)(\lambda_-+\theta)}e^{\frac{\mu_-\theta}{\lambda_-+\theta}z}\int\limits_{y=c}^{z}\widetilde{K}(\theta\,|\,y)e^{-\frac{\mu_-\theta}{\lambda_-+\theta}y}\int\limits_{x=0}^{y-z}e^{-\frac{[(\lambda_-+\theta)\mu_++\mu_-\theta]}{\lambda_-+\theta}x}dxdy$$

$$+\frac{\lambda_+\lambda_-\mu_+}{(\lambda_++\theta)(\lambda_-+\theta)}e^{\frac{\mu_-\theta}{\lambda_-+\theta}z}\int\limits_{y=c}^{d}\widetilde{K}(\theta\,|\,y)e^{-\frac{\mu_-\theta}{\lambda_-+\theta}y}\int\limits_{x=y-z}^{y-c}e^{-\frac{[(\lambda_-+\theta)\mu_++\mu_-\theta]}{\lambda_-+\theta}x}dxdy$$

$$+\frac{\lambda_+\mu_+}{\lambda_++\lambda_-+\theta}e^{-\mu_+z}\int\limits_{y=z}^{d}\widetilde{K}(\lambda_-+\theta\,|\,y)e^{-\mu_+y}dy+$$

$$+\frac{\lambda_-\mu_-}{\lambda_++\lambda_-+\theta}e^{-\mu_-z}\int\limits_{y=c}^{z}\widetilde{K}(\lambda_++\theta\,|\,y)e^{\mu_-y}dy\,.$$

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1. Э.М.Нейманов, Е.А.Ибаев. Преобразования Лапласа-Стильтеса распределения процесса полумарковского блуждания с отражающим экраном в нуле. // Актуальные проблемы математики и информатики тезисы международной конференции, посвященной 90-летию со дня рождения Гейдара Алиева, май 29-31, – 2013, – Баку, – с. 183-185
2. Э.М.Нейманов, Т.И.Насирова, Е.А.Ибаев. Преобразования Лапласа-Стильтеса распределения процесса полумарковского блуждания с отражающим экраном в нуле. // Проблемы управления и информатики Выпуск 1. Украина, – 2015, – с. 97-104. http://jais.org.ua/zhurnal-1.html
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4. E.M.Neymanov. Explicit Form of Laplace-Stieltjes Transform of Joint Distribution of the First PassageTime of Some Level “a”(a>0) and Overshoots Across this Level by a Complex Semi-Markov Walk Process with Reflecting Screen at Zero. // Caspian Journal of applied mathematics, ecology and economics, – Baku, vol. 4, No 1, – 2016, – pp.89-100. http://cjamee.org/wp-content/uploads/2018/09/10.pdf
5. Э.М.Нейманов, Т.И. Насирова, У.Я. Керимова . Интегральное Уравнение с запаздывающим аргументом в процессах полумарковского блуждания. // Journal of Contemporary Applied Mathematics, vol. 7, No 1, – 2017, June, – pp. 9-13 http://journalcam.com/wp-content/uploads/2017/12/2.pdf
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8. Э.М.Нейманов, Т.И. Насирова. Составление уравнения для эрланговского распределения второго порядка в сложном процессе полумарковского блуждания с отражающим экраном в нуле. // "İnformasiya sistemləri və texnologiyalar: nailiyyətlər və perspektivlər" mövzusunda beynəlxalq elmi konfrans, noyabr 15-16, – 2018, – Sumqayıt, – pp. 299.
9. Э.М.Нейманов. Об средном значении числа скачков полумарковского блуждания, при котором процесс впервые достигает уровня нуль // Azərbaycan Mühəndislik Akademiyasının Xəbərləri, – 2019, Volume 3, Number 2; – səh. 79-83

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