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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

NUMERICAL SOLUTION TO THE PROBLEMS OF OPTIMIZATION OF LOCATIONS OF CONTROL AND MEASUREMENT POINTS IN CONTROL SYSTEMS WITH FEEDBACK

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GENERAL DESCRIPTION OF WORK

The relevance and elaboration degree of the topic. It is well known that the problems of optimal control for objects with distributed parameters with feedback have been studied much less in contrast to objects with lumped parameters. The first reason is the complexity of technical implementtation of systems which require information about the current state of the object (process) at all its points. The second is the existence of problems related to the solution of both structural and parametric identification problems of mathematical models for controlled objects, the developing efficient numerical methods and algorithms for solving corresponding mathematical problems.

The end of the XIX – beginning of the XX century as a result of research were developed high-precision measuring devices for the controlling industrial processes and technical objects. In this regard, well-known researchers J.K. Maxwell, E.J. Raus, I.A. Vyshnegradskii, A. Hurvitz, A.M. Lyapunov and other scientists and engineers have made significant contributions to science. With the development of computational and measurement research tools in connection with the problems in the field of rocket development, L.S. Pontryagin, R.E. Bellman, A.M. Letov and other scientists the results of their research in the field of control systems with feedback for objects with lumped parameters described by ordinary differential equations have found wide applications.

However, in recent years, research was actively conducted to extend approaches and methods available for systems with lumped parameters to control over objects with distributed parameters, including feedback control described by special derivative differential equations. The results of the research were used in the design of control and regulation systems for both technical objects and complex industrial processes. Despite certain achieved successes, control synthesis problems for objects with distributed parameters have not yet received widespread application. This is due both to problems of theoretical nature (e.g., due to the study of controllability, observability, development of efficient numerical methods for optimizing the control in the corresponding processes) and with the problems of technical implementation of control systems for these objects with feedback (due to spatial extent, inability to obtain sufficiently up-to-date and accurate information about the current state from all points of the object, as well as the inability to timely implement their control influences distributed in all or some of the points of the object, and other factors). Therefore, it is important to conduct research in this area at this time. At present, automatic control and regulation systems are being developed or are already operating for many objects with distributed parameters using various known principles, computational methods, and technical means of telemechanical control.

In recent years, due to the development of information, computer technologies and the high-precision control and measurement echnologies, there has been a rise in interest in the creation of automatic control and regulation systems for complicated distributed parameter objects described by various types of functional equations with initial-boundary conditions.

It is known that in optimal control theory, program control is sought as a time-dependent function for dynamic systems. These types of control problems describe and is used to solve practical problems in a relatively narrow range, such as space flight or rocket direction control. However, there are many factors and influences that hinder the better application of optimal control theory for certain problems. For example, in many cases there are unavoidable uncertainties due to the inaccuracy of the initial conditions, the very complexity, or sometimes the impossibility of accurately defining the parameters of the model. It is obvious that, the need to establish a control strategy in advance is highly undesirable. For this reason, it is more natural for engineers to choose control as a function of feedback (synthesis) as a function of the current state of the system.

In some optimal control synthesis problems, the phase state of the process is expressed by both “point-wise” or “integral” loaded differential equations for time or space variables, boundary conditions are brought to the initial-boundary problems given by non-local separated intermediate conditions. The description of a large number of technological processes brought to non-local problems by differential equations loaded with partial derivatives has attracted the attention of many researchers working in this field in recent years. Examples of such processes are the synthesis with feedback control, the heating of the plate installed on measuring devices with point-wise heat sources, and the dampinging of the oscillations of the membrane, which becomes oscillating under the influence of certain external blows, with point-wise influence.

Much work has been devoted to the optimal control of the lumped parameter systems, both theoretically and numerically aspects. From these, in works of F.M. Kirillova, L.S. Pontryagin, R.E. Bellman, V.Q. Boltyansky, N.N. Krasovsky, F.P. Vasiliev, R.F. Gabasov, R.P. Fedorenko, and in our republic M.C. Mardanov, K.B. Mansimov, T.Q. Malikov, I.G. Mammadov, Y.A. Sharifov, Sh.F. Maharramov and others necessary and sufficient conditions have been obtained for optimal control problems considered in their work. Obtaining the necessary conditions and the development of numerical methods using them Y.Q. Yevtuşenko, N.N. Moiseev, A.A. Abramov, O.O. Vasilieva, in our republic F.A. Aliyev, M.M. Mutallimov, K.R. Aida-zade, V.M. Abdullaev, A.B. Rahimov and the work of other scientists can be noted.

In most practical problems, many processes are described by distributed parameter systems. Many scientists have studied the optimal control problems for these systems. From these the theory of quality of optimal control works of J.L. Lions, V.Q. Boltyansky, F.P. Vasiliev, A.İ. Egorov, A.D. Isgandarov, K.Q. Hasanov, H.F. Guliyev, K.B. Mansimov, F.G. Feyziyev, M.H. Yagubov, M.A. Sadıqov, R.Q. Tagıyev, S.S. Hakhıyev, Sh. Sh. Yusubov, E.N. Mahmudov, and development of numerical methods F.A. Aliev, K.R. Aida-zade, V.M. Abdullaev, Y.R. Ashrafova, A.B. Rahimov, S.Z. Guliyev, J.A. Asadova, S.G. Talibov and the work of other scientists can be noted.

Many scientists have paid attention at their researches with the widely applied use of feedback optimal control systems, where we can show researches by V.I. Utkin, T.K. Sirazetdinov, A. I. Egorov, A.G. Butkovsky, B.T. Polyak and other scientists. This direction was also paid to the attention of Azerbaijani scientists and many studies were carried out. From these wee can show the work of K.R. Aidazade, V.M. Abdullaev, S.Z. Guliev, Q.A. Rustamov and many other scientists.

Despite the fact that computer technology is developing very rapidly, that there are many problems in computational mathematics, the development of more accurate and faster the development of numerical methods remains relevant. For example, the system of ordinary differential equations, given on non-local conditions, is considered one of the most important classes of computational mathematics. In these problems, the development of numerical solution schemes by approximation of derivatives from high-precision order in the methods of numerical solution of differential equations is of practical importance. For the development of such numerical solution schemes we can especially mention the works of G.Y. Mehdieva, V.R. Ibrahimov, K.R. Aida-zade, V.M. Abdullayev and other scientists.

Object and subject of research. The object of research of the dissertation is mainly the control of distributed parameter systems in relation to the lumped (point-wise) sources. The subject of the research is the first-order effective numerical optimization methods for the synthesis of lumped control sources and measurement points with which feedback is conducted, as well as linear feedback parameters.

The main goal and problems of research. The main objective of the work is as follows:

1. Development of numerical solutions to the problems of synthesis of feedback control systems in distributed parameter systems.
2. Obtaining the gradient of the objective function for the optimal coordinates of the locations of the measurement points and the parameters of linear feedback in the case of extinguishing the oscillations of the elastic membrane, heating a homogeneous rod or a thin plate.
3. Obtaining the gradient of the objective function for the optimal coordinates of the point-wise of impact of the lumped vibration dampers or moments of impact time in the case of extinguishing the vibrations of the membrane.
4. Obtaining gradient of the objective function for the optimal coordinates of the points of point-wise heat sources in the heating

of plate.
5. Consideration of the synthesis of optimal control as a "group" when the initial values of rod or plate heating, or initial membrane oscillation influences on the oscillation process are not known, but the sets in which their values are included and the distribution functions in these sets are given.
6. Development of numerical methods with the application of high-order approximation schemas of linear ordinary differential equations with non-local intermediate conditions.
7. Approximation of the pulse effects in lumped parameter systems or lumped (point-wise) sources in distributed parameter systems with a continuous smooth function, described by one-dimensional or two-dimensional $\delta(x)$-function of Dirac.
8. Development of software packages based on algorithms and numerical methods for computer experiments.

The main research methods. In the dissertation loaded partial differential equations, optimal control, numerical methods of finite-dimensional optimization, numerical methods for initial-boundary value problems for both point-wise and integral meaning loaded partial derivative equations, specially for two-dimensional space variables changing direction method, linear ordinary differential equations with non-local intermediate conditions, as well as object-oriented programming languages used to solve very quickly extremal problems and to create software packages, were used.

The main provisions to be defended are follows:
1. Numerical methods of control and measurement point optimization problems in feedback optimal control problems;
2. Solution of the problem of control synthesis in the calming of oscillations when the forces and points of influences of the initial oscillations of the membrane are given inaccurately;
3. Solution the problem of synthesis of control in heating processes when the initial condition of the plate or ambient temperature is given inaccurately;
4. High-order numerical methods of systems of linear ordinary differential equations given by non-local intermediate conditions;
5. Approximation of lumped (point-wise) sources described by $\delta(x)$-

function of Dirac with continuous, smooth functions;
6. Developing of software for conducting computer experiments with the application of the methods of the conjugate gradient, gradient projection, penalty functions methods to use the obtained first-order necessary conditions.

The scientific novelty. The main scientific innovations of the work are:

1. The gradient of the objective function of the synthesized parameters in the optimal control problems with feedback of distributed parameter objects are obtained.
2. The gradient of the objective function were obtained for the optimization of the coordinates of the location of the lumped sources and a given number of measurement points in the feedback control of the system in the distributed parameter objects.
3. When the initial conditions of the controlled systems or the values of the environmental influences are not known in advance, but given their possible set of values and the distribution functions of the elements included in this set, the optimality conditions for feedback parameters are obtained.
4. Numerical methods for linear ordinary differential equations with non-local intermediate conditions have been developed.
5. Schemes of approximation of one-dimensional and two-dimensional Dirac $\delta(x)$-functions in the solution of initial-boundary problems with ordinary and partial derivative differential equations using grid methods have been proposed and investigated.

Theoretical and practical value of the study. In the research, in theoretically the first-order necessary conditions for the synthesis of feedback optimal control distributed parameter dynamic systems were obtained, which these results can be used in practical engineering optimal control and regulation problems. The proposed approaches can be used in the design of control systems and in the control and regulation of many other distributed parameter systems and objects.

Approbation of the work. The main results of the work were presented in the following various international scientific conferences, both domestic and foreign, in personal or online form, as well as in many scientific seminars.

- **in international conferences:**

“Прикладная математика и фундаментальная информатика” (RF, Omsk, 2017, 2018, 2019, 2020), “Актуальные проблемы прикладной математики и физики” (RF, Nalchik, 2017, 2018), “Математика, ее приложения и математическое образование” (RF, Ulan-Ude, Baykal, 2017), “Дифференциальные уравнения и смежные проблемы” (RF, Samara, 2017), “International Conference on Optimization Methods and Applications (OPTIMA-2017, 2018, 2019, 2020)” (Montenegro, Petrovac), “Control and Optimization with Industrial Applications (COIA- 2018, 2020)” (Bakı), “Modern Problems of Mathematics and Mechanics” beynalxalq konfrans (Bakı, 2019).

- **at scientific seminars:**

Reports on the topics “Qızdırılma prosesinin sərhəd idarəetməsinin sintezi məsələsinin tədqiqi”, “Optimization and control of placements of point dampers on the plate” (Bakı, 2017), “Lövhənin qızdırılması prosesində nöqtəvi ölçmələr və toplanmış istilik mənbələrinin yerləşməsinin optimallaşdırılması” (Bakı, 2018) were made at scientific seminars held at the “Optimal control” department of the Institute of Mathematics and Mechanics of ANAS

Publications. 35 scientific works on the dissertation were published, of which 13 articles, including 11 in foreign countries, 22 conference materials and theses, most of which were presented at influential conferences abroad and in our country. 5 articles are included in the Scopus database and 3 articles are included in the international database of Clarivate Analytics Web of ScienceTM Core Collection.

Institution where the dissertation work was executed. Institute of Control Systems of Azerbaijan National Academy of Science.

Structure and volume of the dissertation. The dissertation consists of 142 pages, 212881 characters, 7 tables and 10 figures, introduction, four chapters, the main result of the work, list of 113 references used, appendix with the source code of the software.

CONTENT OF THE WORK

The **introduction** provides detailed information on the relevance of the problems considered, the obtained scientific innovations, the theoretical and practical importance of the research, its approbation.

The first chapter considers the problems of calming the oscillations processes of the thin elastic membrane in the optimal control of with lumped impact with both of the program and feedback in distributed parameter systems. In feedback control, the mode of operation of the stabilizers is determined linearly by measuring the condition of the membrane in the neighbourhood of the measurement points.

Paragraph 1.1 sets out the optimal control of the membrane oscillation process.

$$u_{tt}(x,t) = a^2\mathcal{L}u(x,t) - \lambda u_t(x,t) + p(x,t) + \quad (1)$$

$$+\sum_{i=1}^{N}\vartheta_i(t)\delta(x;\mathcal{O}_{\varepsilon_x}(\eta^i))\,,\ \ x = (x_1,x_2) \in \Omega,\ \ t \in (0,T],$$

$$u(x,0) = \varphi_0(x),\ \ u_t(x,0) = \varphi_1(x),\ \ x \in \Omega, \quad (2)$$

$$u(x,t) = \varphi_2(x,t),\ \ x \in \Gamma,\ \ t \in (0,T], \quad (3)$$

here, $u(x,t)$ determines state of membrane at moment t and on point $x \in \Omega$; $a^2 > 0$, $\gamma \geq 0$ are given values; $\mathcal{L} = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$ two-dimentional Laplas operator; $\Omega \subset \mathrm{R}^2$ is convex area with given bound Γ. $p(x,t) \in L_2(\Omega \times [0,T])$, $\varphi_0(x) \in L_2(\Omega)$, $\varphi_1(x) \in L_2(\Omega)$, $\varphi_2(x,t) \in L_2(\Gamma \times [0,T])$ are defined given functions; N is number of controled stabilizers. $\vartheta(t) = \left(\vartheta_1(t),\ldots,\vartheta_N(t)\right) \in L_2^N[0,T]$ is vector function which determines the power of lumped stabilizers in sufficient small ε_x neighbourhood of locations of $\eta = (\eta^1,\ldots,\eta^N)$, $\eta^i = \left(\eta_1^i,\eta_2^i\right) \in \Omega$, $i = 1,2,\ldots,N$. T is control time duration.

The function $\delta(x;\mathcal{O}_\varepsilon(\tilde{\eta}))$ which is continuous differentiable for $x \in \Omega$ is determines the distribution of source power around of point $\tilde{\eta} \in \Omega$ in $\mathcal{O}_{\varepsilon_x}(\tilde{\eta})$ and has following features:

$$\delta(x;\mathcal{O}_{\varepsilon_x}(\tilde{\eta}))\begin{cases} \geq 0, & \text{when } x \in \mathcal{O}_{\varepsilon_x}(\tilde{\eta}), \\ = 0, & \text{when } x \notin \mathcal{O}_{\varepsilon_x}(\tilde{\eta}), \end{cases}$$

$$\iint_{\Omega} \delta(x;\mathcal{O}_{\varepsilon_x}(\tilde{\eta}))\,dx = \iint_{\mathcal{O}_{\varepsilon_x}(\tilde{\eta})} \delta(x;\mathcal{O}_{\varepsilon_x}(\tilde{\eta}))\,dx = 1,\ \ \tilde{\eta} \in \Omega.$$

The problem is to determine the location of the control influences η and the power of controls $\vartheta(t)$ that meet certain technological limitations,

$$\eta^i \in \Omega_i \subset \Omega,\ \ i = 1,2,\ldots,N, \tag{4}$$

$$\underline{\vartheta}_i \le \vartheta_i(t) \le \overline{\vartheta}_i,\ \text{almost everyone}\ t \in [0,T], \tag{5}$$

get the following functional minimum value in the shortest T time.

$$\mathcal{J}_T(\vartheta,\eta) = \alpha_1 \iint_{\Omega} [u(x,T) - U_1(x)]^2\,dx + \tag{6}$$

$$+\alpha_2 \iint_{\Omega} [u_t(x,T) - U_2(x)]^2\,dx\,.$$

Here, $U_1(x)$, $U_2(x)$ are the given functions, $\alpha_1 > 0$, $\alpha_2 > 0$ are appropriate weight coefficients.

In paragraph 1.2 the necessary conditions for the optimality of the control parameters of the function (6) are obtained.

Theorem 1. In conditions of (1)–(5) for each duration of time T η control point locations and $\vartheta = \vartheta(t)$ control influences for the functional (6) following formulas for component of gradient are true:

$$\operatorname{grad}_{\vartheta_i(t)}\mathcal{J}_T(\vartheta,\eta) = -\iint_{\mathcal{O}_{\varepsilon_x}(\eta^i)} \psi(x,t)\delta(x;\mathcal{O}_{\varepsilon_x}(\eta^i))\,dx,$$

$$\operatorname{grad}_{\eta_s^i}\mathcal{J}_T(\vartheta,\eta) = -\int_0^T \iint_{\mathcal{O}_{\varepsilon_x}(\eta^i)} \vartheta_i(t)\psi_{x_s}(x,t)\delta(x;\mathcal{O}_{\varepsilon_x}(\eta^i))dx\,dt,$$

$i = 1,2,\ldots,N$, $s = 1,2$. Here function $\psi(x,t)$ is the solution of following adjoint boundary value problem::

$$\psi_{tt}(x,t) = a^2\mathcal{L}\psi(x,t) + \lambda\psi_t(x,t),\ \ \ x \in \Omega,\ \ t \in [0,T),$$
$$\psi(x,T) = -2\alpha_2[u_t(x,T) - U_2(x)],\ \ x \in \Omega,$$
$$\psi_t(x,T) = 2\alpha_1[u(x,T) - U_1(x)] + \lambda\psi(x,T),\ \ x \in \Omega,$$
$$\psi(x,t) = 0,\ \ x \in \Gamma,\ \ t \in [0,T).$$

Theorem 2. If $(\vartheta^*(t),\eta^*)$ is the local minimum of the problem (1)–(3), (4), (5), (6), then for the satisfing conditions (4) and (5) arbitrary parameters $(\vartheta(t),\eta)$

$$\sum_{i=1}^{N}\int_{0}^{T}\iint\limits_{O_{\varepsilon_x}(\eta^{i*})}\psi(x,t)\delta(x;O_{\varepsilon_x}(\eta^{i*}))\big(\vartheta_i(t)-\vartheta_i^*(t)\big)\,dx\,dt+$$

$$+\sum_{i=1}^{N}\sum_{s=1}^{2}\int_{0}^{T}\iint\limits_{O_{\varepsilon_x}(\eta^{i*})}\vartheta_i^*(t)\psi_{x_s}(x,t)\delta(x;O_{\varepsilon_x}(\eta^{i*}))\big(\eta_s^i-\eta_s^{i,*}\big)dx\,dt\le 0$$

it is necessary satisfing of inequality.

Paragraph 1.3 we consider the problem of suppressing the transverse vibrations of a thin homogeneous membrane of given shape restrained along the boundary. The vibrations are assumed to be generated by external disturbances at the initial time in a neighborhood of the membrane points θ^ν, $\nu=1,2,\ldots,L$. The vibrations are suppressed by stabilizers affecting neighborhoods of the membrane points η^i, $i=1,2,\ldots,N_c$ in neighborhoods of given time moments τ_s, $s=1,2,\ldots,N_t$. The operation modes of the stabilizers are determined using the current membrane displacements measured by sensors placed in a neighborhood of the points ξ^j, $j=1,2,\ldots,N_o$.

For $t>0$ this process can be described by the initial-boundary value problem:

$$u_{tt}(x,t)=a^2\mathcal{L}u(x,t)-\lambda u_t(x,t)+ \tag{7}$$

$$+\sum_{s=1}^{N_t}\delta(t;O_{\varepsilon_t}(\tau_s))\sum_{i=1}^{N_c}\vartheta_s^i\delta(x;O_{\varepsilon_x}(\eta^i)),\ \ x=(x_1,x_2)\in\Omega,$$

$$u(x,0)=0,\ \ u_t(x,0)=\sum_{\nu=1}^{L}q^\nu\delta(x;O_{\varepsilon_x}(\theta^\nu)),\ \ x\in\Omega, \tag{8}$$

$$u(x,t)=0,\ \ x\in\Gamma, \tag{9}$$

here, the function $u(x,t)$ determines the displacement of the membrane at the point $x\in\Omega\subset \mathrm{R}^2$ at the time moment t; a^2, $\lambda\ge 0$ are given constants; Γ is the almost verywhere smooth boundary of the domain Ω; q^ν is the intensity of the ν-th external disturbance lumped in an neighborhood of the membrane point $\theta^\nu=(\theta_1^\nu,\theta_2^\nu)\in\Omega$, $\nu=1,2,\ldots,L$; where L is the number of such points; $\vartheta=\big(\vartheta_1^1,\ldots,\vartheta_1^{N_c},\ldots,\vartheta_{N_t}^1,\ldots,\vartheta_{N_t}^{N_c}\big)\in \mathrm{R}^{N_tN_c}$ is the vector determining control functions of stabilizers on the neighborhood of the points $\eta^i=$

$\left(\eta_1^i,\eta_2^i\right) \in \Omega,\ i = 1,2,\ldots,N_c,\ \eta = (\eta^1,\eta^2,\ldots,\eta^{N_c}),\ \tau = \left(\tau_1,\tau_2,\ldots,\tau_{N_t}\right)$ are given moments of time in the neighborhood of which vibration suppression is applied, where, $\tau_s > \tau_{s-1} > 0$, $s = 1,2,\ldots,N_t$, $\tau_0 = 0$, $\tau_{N_t} = T_f$; N_t is the number of time moments and T_f is a given duration of the control process.

Suppose that the power q^ν of the external sources and their location points θ^ν, $\nu = 1,2,\ldots,N_b$ are known inexactly, but their possible sets Q^ν and Θ^ν, and the distribution functions $\rho_{Q^\nu}(q) \geq 0$, $\rho_{\Theta^\nu}(\theta) \geq 0$ are given in these sets.

The intensities ϑ_s^i of the control influences and their location points η^i are the parameters to be optimized in the considered control process of vibration extinguishing. They satisfy the constraints:

$$\underline{\vartheta^i} \leq \vartheta_s^i \leq \overline{\vartheta^i},\ \ i = 1,2,\ldots,N_c,\ \ s = 1,2,\ldots,N_t, \tag{10}$$

$$\eta^i \in \mathcal{O}_{\varepsilon_x}\left(\eta^i\right) \subset \Omega_c^i \subset \Omega,\ \ i = 1,2,\ldots,N_c, \tag{11}$$

In (11) Ω_c^i are given closed subdomains in which stabilizers can be placed; $\underline{\vartheta^i}$ and $\overline{\vartheta^i}$ are given, $i = 1,2,\ldots,N_c$.

Let us determine the current values of the influences as follows, which determine the feedback on the state of the membrane at the measuring points of the control effects:

$$\hat{u}_s^j = \int\limits_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint\limits_{\mathcal{O}_{\varepsilon_x}(\xi^j)} u(\hat{x},\hat{t})\delta(\hat{x};\mathcal{O}_{\varepsilon_x}(\xi^j))\delta(\hat{t};\mathcal{O}_{\varepsilon_t}(\tau_s))\, d\hat{x}\, d\hat{t}\,, \tag{10}$$

$$\vartheta_s^i = \sum_{j=1}^{N_o} k_i^j \left[\int\limits_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint\limits_{\mathcal{O}_{\varepsilon}(\xi^j)} u(\hat{x},\hat{t})\delta(\hat{x};\mathcal{O}_{\varepsilon_x}(\xi^j))\delta(\hat{t};\mathcal{O}_{\varepsilon_t}(\tau_s))\, d\hat{x}\, d\hat{t} - z_i^j\right], \tag{11}$$

$i = 1,2,\ldots,N_c$, $j = 1,2,\ldots,N_o$, $s = 1,2,\ldots,N_t$. Here, $k = ((k_i^j))$ is the gain matrix; $z = ((z_i^j))$, where z_i^j is the nominal membrane displacement at the point ξ^j relative to the stabilizer placed at the point η^i, $i = 1,2,\ldots,N_c$, $j = 1,2,\ldots,N_o$; k, z are the feedback parameters to be optimized.

Substituting formula (11) into equation (7) yields:

$$u_{tt}(x,t) = a^2\mathcal{L}u(x,t) - \lambda u_t(x,t) + \sum_{s=1}^{N_t} \delta(t;\mathcal{O}_{\varepsilon_t}(\tau_s)) \sum_{i=1}^{N_c} \delta(x;\mathcal{O}_{\varepsilon_x}(\eta^i)) \times$$

$$\times \sum_{j=1}^{N_o} k_i^j \left[\int\limits_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint\limits_{\mathcal{O}_{\varepsilon_x}(\xi^j)} u(\hat{x},\hat{t})\delta(\hat{x};\mathcal{O}_{\varepsilon_x}(\xi^j))\delta(\hat{t};\mathcal{O}_{\varepsilon_t}(\tau_s))\, d\hat{x}\, d\hat{t} - z_i^j \right], \quad (14)$$

$$x \in \Omega.$$

Devices for measuring the condition of a membrane cannot be placed at all its points, only at certain altoblasts:

$$\xi^j \in \mathcal{O}_{\varepsilon_x}\left(\xi^j\right) \subset \Omega_o^j \subset \Omega, \ \ j = 1,2, \dots, N_o, \quad (15)$$

and subdomains of the locations of the stabilizer and the of the measuring points may not intersect according to the conditions (11) and (15), i.e.

$$\Omega_c^i \cap \Omega_o^j = \emptyset, \quad i = 1,2, \dots, N_c, \quad j = 1,2, \dots, N_o.$$

The purpose of the matter under consideration is to control the process of calming the oscillations of the membrane $k \in \mathrm{R}^{N_c N_o}$, $z \in \mathrm{R}^{N_c N_o}$ feedback parameters, ξ measurement və η consists of determining the optimal values of the locations of the soothing points. Finite-dimensional $y = (k, z, \xi, \eta)$ total size of synthesized parameter vector is equal to $N = 2(N_c N_o + N_c + N_o)$, that is, $y \in \mathrm{R}^N$.

Enter quality criteria for the control parameters defined by the functions:

$$\mathcal{J}(y) = \int\limits_Q \iint\limits_\Theta I(y;q,\theta)\rho_\Theta(\theta)\rho_Q(q)\, d\theta\, dq\,, \quad (16)$$

$$I(y;q,\theta) = \int\limits_{T_f}^{T_1} \iint\limits_\Omega \mu(x)[u(x,t;y,q,\theta)]^2\, dx\, dt + \Re(y;\varepsilon), \quad (17)$$

$$\Re(y;\varepsilon) = \varepsilon_1 \left\| k - \hat{k} \right\|^2_{\mathrm{R}^{N_c N_o}} + \varepsilon_2 \| z - \hat{z} \|^2_{\mathrm{R}^{N_c N_o}} +$$

$$+ \varepsilon_3 \left\| \xi - \hat{\xi} \right\|^2_{\mathrm{R}^{2N_o}} + \varepsilon_4 \| \eta - \hat{\eta} \|^2_{\mathrm{R}^{2N_c}},$$

here, $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4)$, $\varepsilon_i \geq 0$, $i = 1,2, \dots ,4$, $\hat{k} \in \mathrm{R}^{N_c N_o}$, $\hat{z} \in \mathrm{R}^{N_c N_o}$, $\hat{\xi} \in \mathrm{R}^{2N_o}$, $\hat{\eta} \in \mathrm{R}^{2N_c}$ are regulation parameters.

We use penalty functions taking into account the constraints (10) in in the optimization of the feedback parameters y. (16) To the integral (17) functionality of the objective functionality, the addition of a penalty term:

$$\tilde{\mathcal{J}}(y) = \int\limits_Q \iint\limits_\Theta \tilde{I}(y;q,\theta)\rho_\Theta(\theta)\rho_Q(q)\,d\theta\,dq\,, \tag{18}$$

$$\tilde{I}(y;q,\theta) = I(y;q,\theta) + \mathcal{R}G(y). \tag{19}$$

Here, we used the following notation:

$$G(y) = \sum_{s=1}^{N_t}\sum_{i=1}^{N_c}[g_i^+(\tau_s;y)]^2\,,$$

$$g_i^+(\tau_s;y) = \begin{cases} 0, & g_i(\tau_s;y) \le 0, \\ g_i(\tau_s;y), & g_i(\tau_s;y) > 0, \quad i = 1,2,\ldots,N_c, \;\; s = 1,2,\ldots,N_t. \end{cases}$$

In (19) the parameter $\mathcal{R} > 0$ is the penalty coefficient, $\mathcal{R} \to +\infty$.

Teorem 3. In the problem of (14), (8)–(11), (15), (18), (19) the components of the gradient of functional (18) with respect to the parameters $y = (k, z, \xi, \eta)$ of linear feedback (12), (13) are given by the formulas:

$$\frac{\partial\tilde{\mathcal{J}}(y)}{\partial k_i^j} = \int\limits_Q \int\limits_\Theta \Bigg\{ -\sum_{s=1}^{N_t} \Bigg[\int\limits_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint\limits_{\mathcal{O}_{\varepsilon_x}(\xi^j)} u(\hat{x},\hat{t})\delta(\hat{x};\mathcal{O}_{\varepsilon_x}(\xi^j))\delta(\hat{t};\mathcal{O}_{\varepsilon_t}(\tau_s))\,d\hat{x}\,d\hat{t} - z_i^j \Bigg] \times$$

$$\times \Bigg[\int\limits_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint\limits_{\mathcal{O}_{\varepsilon_x}(\eta^i)} \psi(x,t)\delta(x;\mathcal{O}_{\varepsilon_x}(\eta^i))\delta(t;\mathcal{O}_{\varepsilon_t}(\tau_s))\,dx\,dt +$$

$$+ 2\mathcal{R}g_i^+(\tau_s;y)\,\mathrm{sgn}\left(g_i^0(\tau_s;y)\right) \Bigg] + 2\varepsilon_1\left(k_i^j - \hat{k}_i^j\right) \Bigg\} \rho_\Theta(\theta)\rho_Q(q)\,d\theta\,dq,$$

$$\frac{\partial\tilde{\mathcal{J}}(y)}{\partial z_i^j} = \int\limits_Q \iint\limits_\Theta \Bigg\{ k_i^j \sum_{s=1}^{N_t} \Bigg[\int\limits_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint\limits_{\mathcal{O}_{\varepsilon_x}(\eta^i)} \psi(x,t)\delta(x;\mathcal{O}_{\varepsilon_x}(\eta^i))\delta(t;\mathcal{O}_{\varepsilon_t}(\tau_s))\,dx\,dt +$$

$$+2\mathcal{R}g_i^+(\tau_s;y)\,\mathrm{sgn}\left(g_i^0(\tau_s;y)\right) \Bigg] + 2\varepsilon_2\left(z_i^j - \hat{z}_i^j\right) \Bigg\} \rho_\Theta(\theta)\rho_Q(q)\,d\theta\,dq,$$

$$\frac{\partial\tilde{\mathcal{J}}(y)}{\partial \xi_\gamma^j} = \int\limits_Q \iint\limits_\Theta \Bigg\{ -\sum_{s=1}^{N_t} \int\limits_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint\limits_{\mathcal{O}_{\varepsilon_x}(\xi^j)} u_{\hat{x}_\gamma}(\hat{x},\hat{t})\delta(\hat{x};\mathcal{O}_{\varepsilon_x}(\xi^j))\delta(\hat{t};\mathcal{O}_{\varepsilon_t}(\tau_s))\,d\hat{x}\,d\hat{t} \times$$

$$\times \sum_{i=1}^{N_c} k_i^j \Bigg[\int\limits_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint\limits_{\mathcal{O}_{\varepsilon_x}(\eta^i)} \psi(x,t)\delta(x;\mathcal{O}_{\varepsilon_x}(\eta^i))\delta(t;\mathcal{O}_{\varepsilon_t}(\tau_s))\,dx\,dt +$$

$$+ 2\mathcal{R}g_i^+(\tau_s;y)\operatorname{sgn}\left(g_i^0(\tau_s;y)\right)\Bigg] + 2\varepsilon_3\left(\xi_\gamma^j - \hat{\xi}_\gamma^j\right)\Bigg\}\rho_\Theta(\theta)\rho_Q(q)\,d\theta\,dq,$$

$$\frac{\partial \tilde{J}(y)}{\partial \eta_\gamma^i} = \int_Q \iint_\Theta \Bigg\{-\sum_{s=1}^{N_t} \int_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint_{\mathcal{O}_{\varepsilon_x}(\eta^i)} \psi_{x_\gamma}(x,t)\delta(x;\mathcal{O}_{\varepsilon_x}(\eta^i))\delta(t;\mathcal{O}_{\varepsilon_t}(\tau_s))\,dx\,dt \times$$

$$\times \sum_{j=1}^{N_o} k_i^j \left[\int_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint_{\mathcal{O}_{\varepsilon_x}(\xi^j)} u(\hat{x},\hat{t})\delta(\hat{x};\mathcal{O}_{\varepsilon_x}(\xi^j))\delta(\hat{t};\mathcal{O}_{\varepsilon_t}(\tau_s))\,d\hat{x}\,d\hat{t} - z_i^j\right] +$$

$$+ 2\varepsilon_4\left(\eta_\gamma^i - \hat{\eta}_\gamma^i\right)\Bigg\}\rho_\Theta(\theta)\rho_Q(q)\,d\theta\,dq,$$

$i = 1,2,\dots,N_c$, $j = 1,2,\dots,N_o$, $\gamma = 1,2$. Here the function $\psi(x,t)$ is a solution of the adjoint initial-boundary value problem:

$$\psi_{tt}(x,t) = a^2\mathcal{L}\psi(x,t) + \lambda\psi_t(x,t) - 2u(x,t;y,q,\theta)\chi_{[T_f,T_1]}(t) +$$

$$+\sum_{s=1}^{N_t}\delta(t;\mathcal{O}_{\varepsilon_t}(\tau_s))\sum_{j=1}^{N_o}\delta(x;\mathcal{O}_{\varepsilon_x}(\xi^j)) \times$$

$$\times \sum_{i=1}^{N_c} k_i^j \left[\int_{\mathcal{O}_{\varepsilon_t}(\tau_s)} \iint_{\mathcal{O}_{\varepsilon_x}(\eta^i)} \psi(\hat{x},\hat{t})\delta(\hat{x};\mathcal{O}_{\varepsilon_x}(\eta^i))\delta(\hat{t};\mathcal{O}_{\varepsilon_t}(\tau_s))\,d\hat{x}\,d\hat{t} +\right.$$

$$\left. + 2rg_i^+(\tau_s;y)\operatorname{sgn}\left(g_i^0(\tau_s;y)\right)\right], \quad x \in \Omega, \quad t \in [0,T_1),$$

$$\psi(x,T_1) = 0, \quad \psi_t(x,T_1) = 0, \quad x \in \Omega,$$
$$\psi(x,t) = 0, \quad x \in \Gamma, \quad t \in [0,T_1).$$

The second chapter the approach to the problem of optimal synthesis of feedback control of distributed parameter objects described by partial differential equations was studed.

Paragraph 2.1 the synthesis of controls in the example of the rod heating process was considered.

Let us consider the problem of controlling the process of successive heating homogeneous identical rods of length l from the same end in succession. The setting of the heating process is given by the following initial-boundary problem:

$$u_t(x,t) = a^2 u_{xx}(x,t) - \lambda_0[u(x,t) - \theta], \quad (20)$$
$$(x,t) \in \Omega = (0,l) \times (0,T],$$
$$u_x(0,t) = \lambda_1[u(0,t) - \vartheta(t)], \ \ t \in (0,T], \quad (21)$$
$$u_x(l,t) = -\lambda_2[u(l,t) - \theta], \ \ t \in (0,T]. \quad (22)$$

Here, $u(x,t)$ is the temperature of the rod at point x at time t; $\theta =$ const is the ambient temperature; $a^2 =$ const is thermal conductivity, λ_0, λ_1 və λ_2 are given coefficients. T is the duration of the process. $\vartheta = \vartheta(t)$ is the temperature of the control source, which is a piecewise continuous function of time and which satisfies technological constraints:

$$\underline{\vartheta} \le \vartheta(t) \le \overline{\vartheta}, \ \ t \in [0,T], \quad (23)$$

here $\underline{\vartheta}$ and $\overline{\vartheta}$ are known.

It is assumed that the initial temperature of the rods was not given in advance. However, the initial temperature value of each rod is constant along its length, and the distribution function $\rho_\Phi(\varphi) \ge 0$ is known for this possible set Φ:

$$u(x,0) = \varphi(x) = \varphi = \text{const} \in \Phi, \ \ x \in [0,l]. \quad (24)$$

The ambient temperature $\theta =$ const may also be set not exactly but rather defined by the set of possible values Θ with a given density function $\rho_\Theta(\theta) \ge 0$:

$$\theta \in \Theta, \ \ t \in [0,T]. \quad (25)$$

Suppose that in L_x points of the rod $\xi_i \in [0,l]$, $i = 1,2,\ldots,L_x$

$$u_i(t) = u(\xi_i,t), \ \ i = 1,2,\ldots,L_x. \quad (23)$$

during the heating process the temperature is measured continuously in time. Here

$$0 \le \xi_i \le l, \ \ i = 1,2,\ldots,L_x, \quad (26)$$

$\vartheta(t)$ the value of the boundary control is determined as the linear feedback of the results obtained from the measurement points based on the current state of the process. If the feedback is as continuous as (26), the control is determined by the following formula:

$$\vartheta(t;y) = \sum_{i=1}^{L_x} k_i[u(\xi_i,t) - z_i], \ \ t \in [0,T], \quad (28)$$

here, z_i is the nominal value of the temperature at the i-th measurement point such that the deviation of the current state at this point affects the control value; k_i are gain coefficients, $i = 1,2,\ldots,L_x$. $k = (k_1,k_2,\ldots,k_{L_x})$, $z = (z_1,z_2,\ldots,z_{L_x})$, $\xi = (\xi_1,\xi_2,\ldots,\xi_{L_x})$, $y = (k,z,\xi)$ below and in (28) we use the following notation.

Substituting (28) into the boundary condition (21),

$$u_x(0,t) = \lambda_1\left(u(0,t) - \sum_{i=1}^{L_x} k_i[u(\xi_i,t) - z_i]\right),\ \ t \in (0,T], \qquad (29)$$

we obtain a nonlocal boundary condition with non-separated intermediate conditions.

When the control of the heating process is carried out with continuous measurements over time (26), we can write the objective functions as follows:

$$\mathcal{J}(y) = \int_{\Theta}\int_{\Phi} I(y;\varphi,\theta)\rho_\Phi(\varphi)\,\rho_\Theta(\theta)d\varphi\, d\theta\,, \qquad (30)$$

$$I(y;\varphi,\theta) = \int_0^l \mu(x)[u(x,T;y,\varphi,\theta) - U(x)]^2 dx + \qquad (31)$$

$$+\varepsilon\|y - \hat{y}\|^2_{R^{3L_x}}.$$

Synthesis of the finite-dimensional parameter vector $y \in R^{3L_x}$ of control (25) with continuous (26) measurements (20), (29), (22), (24), (25), (30), (31) in the solution of the problem taking into account the constraint condition (23) and for the condition (27) for minimization will use with the application of the penalty functions and the gradient projection method.

For simplicity, let's write the condition (23) as a new inequality in equivalent form:

$$g(t;y) = d_1 - \left|\vartheta_{d_0}(t;y)\right| \geq 0,\ \ t \in [0,T], \qquad (32)$$

$$\vartheta_{d_0}(t;y) = d_0 - \vartheta(t;y),\ \ d_0 = \frac{\overline{\vartheta} + \underline{\underline{\vartheta}}}{2},\ \ d_1 = \frac{\overline{\vartheta} - \underline{\underline{\vartheta}}}{2}.$$

We will use the internal penalty function (32) to take into account the conditions (23). In this case, we can write the minimized function (31) as follows:

$$\tilde{I}(y;\varphi,\theta) = I(y;\varphi,\theta) + \mathcal{R}G(y), \qquad (33)$$

$$G(y) = \int_0^T [\min(0, g(t,y))]^2 \, dt,$$

here, $\mathcal{R}$ is the penalty factor and $\mathcal{R} \to +\infty$.

Theorem 4. The components of the gradient of the penalty functional (30), (33) according to the (28) continuous by synthesized feedback parameters of the in the initial-boundary value problem (20), (29), (22), (24), (25) are determined by the following formulas:

$$\frac{\partial \mathcal{J}(y)}{\partial k_i} = \int_\Theta \int_\Phi \left\{ \int_0^T [-\lambda_1 a^2 \psi(0,t)(u(\xi_i,t) - z_i) + 2\mathcal{R}(u(\xi_i,t) - z_i) \times \right.$$

$$\left. \times \operatorname{sgn}(\vartheta_{d_0}(t,y)) \min\big(0, g(t,y)\big)\big] dt + 2\varepsilon\big(k_i - \hat{k}_i\big) \right\} \rho_\Phi(\varphi) \rho_\Theta(\theta) d\varphi \, d\theta,$$

$$\frac{\partial \mathcal{J}(y)}{\partial z_i} = \int_\Theta \int_\Phi \left\{ \int_0^T \big[\lambda_1 a^2 \psi(0,t) k_i - \right.$$

$$\left. - 2\mathcal{R} k_i \operatorname{sgn}(\vartheta_{d_0}(t,y)) \min\big(0, g(t,y)\big)\big] dt + 2\varepsilon(z_i - \hat{z}_i) \right\} \times$$

$$\times \rho_\Phi(\varphi) \rho_\Theta(\theta) d\varphi \, d\theta,$$

$$\frac{\partial \mathcal{J}(y)}{\partial \xi_i} = \int_\Theta \int_\Phi \left\{ \int_0^T [-\lambda_1 a^2 \psi(0,t) k_i u_x(\xi_i,t) + 2\mathcal{R} k_i u_x(\xi_i,t) \times \right.$$

$$\left. \times \operatorname{sgn}(\vartheta_{d_0}(t,y)) \min\big(0, g(t,y)\big)\big] dt + 2\varepsilon\big(\xi_i - \hat{\xi}_i\big) \right\} \rho_\Phi(\varphi) \rho_\Theta(\theta) d\varphi \, d\theta,$$

$i = 1,2,\ldots,L_x$. Here, $\psi(x,t) = \psi(x,t;y,\varphi,\theta)$ for all $x \in (\xi_i, \xi_{i+1})$, $i = 0,1,\ldots,L_x$, in intervals is the solution of the initial-boundary value problem

$$\psi_t(x,t) = -a^2 \psi_{xx}(x,t) + \lambda_0 \psi(x,t),$$
$$x \in (\xi_i, \xi_{i+1}), \;\; i = 0,\ldots,L_x, \;\; t \in [0,T),$$
$$\psi(x,T) = -2\mu(x)\big(u(x,T) - U(x)\big), \;\; x \in [0,l],$$
$$\psi_x(0,t) = \lambda_1 \psi(0,t), \;\; t \in [0,T),$$
$$\psi_x(l,t) = -\lambda_2 \psi(l,t), \;\; t \in [0,T),$$

but, in $\xi_i \in (0,l)$, $i = 1,2,\ldots,L_x$ the following conditions must be met at observation points:

$$\psi_x(\xi_i^+,t) = \psi_x(\xi_i^-,t) - \lambda_1\psi(0,t)k_i + $$
$$+\frac{2k_i\mathcal{R}}{a^2}\operatorname{sgn}\left(\vartheta_{d_0}(t,y)\right)\min(0,g(t,y)),\ \ i = 1,2,\ldots,L_x,$$
$$\psi(\xi_i^+,t) = \psi(\xi_i^-,t),\ \ i = 1,2,\ldots,L_x.$$

In paragraph 2.2 the optimization of control and measurement points, as well as linear feedback parameters, in the synthesis of control in the process of heating a thin plate with the given number of point-wise heat sources is considered.

$$u_t(x,t) = a^2\operatorname{div}(\operatorname{grad}u(x,t)) - \lambda_0[u(x,t) - \theta] + , \qquad (34)$$
$$+\sum_{i=1}^{N_c}\vartheta_i(t)\delta\left(x - \eta^i\right), x = (x_1,x_2) \in \Omega \subset \mathrm{R}^2,\ \ t \in (0,T],$$
$$u(x,0) = \varphi(x) = \mathrm{const} \in \Phi,\ \ x \in \Omega, \qquad (35)$$
$$\frac{\partial u(x,t)}{\partial \mathrm{n}} = \lambda[u(x,t) - \theta],\ \ x \in \Gamma,\ \ t \in (0,T]. \qquad (36)$$

Here, a^2, λ_0, λ are given coefficients; Ω the boundary surrounding the plate Γ, n is internal normal to the plate Γ.

Assume that the initial temperature $\varphi(x)$ of the plate and the ambient temperature $\theta = \mathrm{const}$ are not known exactly, the set of possible values of the initial temperature $\Phi \subset \mathrm{R}$ and in this set the corresponding $\rho_\Phi(\varphi) \geq 0$, the set of possible values of the ambient temperature $\Theta \subset \mathrm{R}$ and distribution function $\rho_\Phi(\varphi) \geq 0$ corresponding to the values from these sets are known.

Suppose that in the process of heating on the plate at N_o points $j = 1, \ldots, N_o$

$$\xi^j = \left(\xi_1^j,\xi_2^j\right) \in \Omega,\ \ \underline{a_1} \leq \xi_1^j \leq \overline{a_1},\ \ \underline{a_2} \leq \xi_2^j \leq \overline{a_2}, \qquad (37)$$
$$j = 1,2,\ldots,N_o,$$

continuously over time

$$u_{\xi^j}(t) = u\left(\xi^j,t\right),\ \ \xi^j \in \Omega,\ \ t \in [0,T],\ \ j = 1,2,\ldots,N_o. \qquad (38)$$

current temperature measurement is performed.

When the measurements are as (38), $\vartheta_i(t)$, $i = 1, \ldots, N_c$, the values of the point-wise control sources are given with linear feedback by the following formula:

$$\vartheta_i(t) = \sum_{j=1}^{N_o} k_i^j \left[u\left(\xi^j,t\right) - z_i^j\right], \;\; t \in [0,T], \;\; i = 1,2,\ldots,N_c. \qquad (39)$$

Here k_i^j is the gain coefficient for source i taking into account the temperature at measuring point j; z_i^j is the nominal value of the plate temperature at measuring point j that should be supported by source i.

We substitute expression (39) into the differential equation (34):

$$u_t(x,t) = a^2 \operatorname{div}(\operatorname{grad} u(x,t)) - \lambda_0[u(x,t) - \theta] + \qquad (40)$$

$$+\sum_{i=1}^{N_c}\sum_{j=1}^{N_o} k_i^j\left[u\left(\xi^j,t\right) - z_i^j\right]\delta\left(x-\eta^i\right), \;\; x \in \Omega, \;\; t \in (0,T],$$

The possible values of control sources are defined as follows:

$$\underline{\vartheta_i} \le \vartheta_i(t) \le \overline{\vartheta_i}, \;\; i = 1,\ldots,N_c, \;\; t \in [0,T], \qquad (41)$$

The points of location of heat sources $\eta = (\eta^1,\ldots,\eta^{N_c})$ must meet the following conditions:

$$\eta^i = \left(\eta_1^i,\eta_2^i\right) \in \Omega, \;\; \underline{a_1} \le \eta_1^i \le \overline{a_1}, \;\; \underline{a_2} \le \eta_2^i \le \overline{a_2}, i = 1,\ldots,N_c. \qquad (42)$$

Optimized parameters in the considered problem $\eta \in \mathrm{R}^{2N_c}$, $\xi \in \mathrm{R}^{2N_o}$, K, $Z \in \mathrm{R}^{N_c N_o}$ are fixed parameters. Total number $n = 2(N_c N_o + N_o + N_c)$ and $y = (K,Z,\xi,\eta) \in \mathrm{R}^n$.

Let's the criterion of control is the following functional:

$$\mathcal{J}(y) = \int_{\Phi}\int_{\Theta} I(y;\varphi,\theta)\rho_\Theta(\theta)\rho_\Phi(\varphi)\,d\theta\,d\varphi\,, \qquad (43)$$

$$I(y;\varphi,\theta) = \iint_{\Omega} \mu(x)[u(x,T;y,\varphi,\theta) - U(x)]^2\,dx + \varepsilon\|y-\hat{y}\|_{\mathrm{R}^n}^2, \qquad (44)$$

Let us denote the solution of the initial-boundary value problem by (40), (35), (36) for any possible vector y, initial condition and θ by the ambient temperature as $u(x,t;y,\varphi,\theta)$. $\hat{y}$, $\varepsilon > 0$ are regularization parameters.

If we replace (39) with (41) for control sources, we get:

$$\underline{\vartheta_i} \le \sum_{j=1}^{N_o} k_i^j\left[u\left(\xi^j,t\right) - z_i^j\right] \le \overline{\vartheta_i}, \;\; t \in [0,T], \;\; i = 1,2,\ldots,N_c. \qquad (45)$$

Let’s enter the following notation

$$g_i(t;y) = \left|g_i^0(t;y)\right| - \frac{\overline{\vartheta_i} - \underline{\vartheta_i}}{2},$$

$$g_i^0(t;y) = \frac{\overline{\vartheta_i} + \underline{\vartheta_i}}{2} - \sum_{j=1}^{N_o} k_i^j\left[u\left(\xi^j,t\right) - z_i^j\right], t \in [0,T], i = 1,2,\ldots,N_c.$$

Then we can write the constraint condition (45) in a simpler form:

$$g_i(t;y) \leq 0, \;\; t \in [0,T], \;\; i = 1,2,\ldots,N_c. \qquad (46)$$

The distance between the location of the sources and the measurement points is not less than d:

$$\left(\xi_1^j - \eta_1^i\right)^2 + \left(\xi_2^j - \eta_2^i\right)^2 \geq d^2, \qquad (47)$$
$$i = 1,2,\ldots,N_c, \;\; j = 1,2,\ldots,N_o.$$

Let N_cN_o in (47) be shown below and add (44)

$$g_{N_c+(i-1)N_o+j}(\cdot\,;y) = d^2 - \left(\xi_1^j - \eta_1^i\right)^2 + \left(\xi_2^j - \eta_2^i\right)^2 \leq 0, \qquad (48)$$
$$i = 1,2,\ldots,N_c, \;\; j = 1,2,\ldots,N_o.$$

The total number of constraint conditions (46) and (48) is equal to $N = N_c(N_o + 1)$.

In order to satisfy the constraints (46) and (48) on the synthesis of the considered y parameters, we will use the external penalty functions by adding a penalty limit to the function (43), (44):

$$\tilde{J}(y) = \int_\Phi \int_\Theta \tilde{I}(y;\varphi,\theta)\rho_\Theta(\theta)\rho_\Phi(\varphi)\, d\theta\, d\varphi\,, \qquad (49)$$

$$\tilde{I}(y;\varphi,\theta) = I(y;\varphi,\theta) + G(y), \qquad (50)$$

$$G(y) = \sum_{i=1}^{N_c} \mathcal{R}_i \int_0^T [g_i^+(t;y)]^2\, dt + \sum_{i=N_c+1}^{N} \mathcal{R}_i[g_i^+(\cdot\,;y)]^2\,.$$

here, $\mathcal{R}_i \to +\infty$, $i = 1,2,\ldots,N$ are penalty coefficients.

Teorem 5. The components of the gradient of the penalty functional (49), (50) according to the (38) continuous by synthesized feedback parameters in the initial-boundary value problem (40), (35), (36), (37), (42) are determined by the following formulas:

$$\frac{\partial \tilde{J}(y)}{\partial k_i^j} = \int_\Phi \int_\Theta \Bigg\{ -\int_0^T \left(\psi(\eta^i,t) + 2\mathcal{R}_i g_i^+(t;y)\,\mathrm{sgn}(g_i^0(t;y))\right) \times$$

$$\times \left[u\left(\xi^j,t\right) - z_i^j\right] dt + 2\varepsilon\left(k_i^j - \hat{k}_i^j\right)\Bigg\} \rho_\Theta(\theta)\rho_\Phi(\varphi)\, d\theta\, d\varphi\,,$$

$$\frac{\partial \tilde{J}(y)}{\partial z_i^j} = \int\limits_\Phi \int\limits_\Theta \Bigg\{ k_i^j \int\limits_0^T \left(\psi(\eta^i,t) + 2\mathcal{R}_i g_i^+(t;y)\,\mathrm{sgn}(g_i^0(t;y))\right) dt +$$

$$+ 2\varepsilon\left(z_i^j - \hat{z}_i^j\right)\Bigg\} \rho_\Theta(\theta)\rho_\Phi(\varphi)\, d\theta\, d\varphi\,,$$

$$\frac{\partial \tilde{J}(y)}{\partial \xi_\gamma^j} = \int\limits_\Phi \int\limits_\Theta \Bigg\{ -\sum_{i=1}^{N_c} k_i^j \int\limits_0^T u_{x_\gamma}\left(\xi^j,t\right)\left(\psi(\eta^i,t) + 2\mathcal{R}_i g_i^+(t;y) \times\right.$$

$$\left.\times\, \mathrm{sgn}(g_i^0(t;y))\right) dt + 4\sum_{i=1}^{N_c} \mathcal{R}_{N_c+(i-1)N_o+j}\left(\eta_\gamma^i - \xi_\gamma^j\right) \times$$

$$\times\, g_{N_c+(i-1)N_o+j}^+(\cdot\,,y) + 2\varepsilon\left(\xi_\gamma^j - \hat{\xi}_\gamma^j\right)\Bigg\} \rho_\Theta(\theta)\rho_\Phi(\varphi)\, d\theta\, d\varphi\,,$$

$$\frac{\partial \tilde{J}(y)}{\partial \eta_\gamma^i} = \int\limits_\Phi \int\limits_\Theta \Bigg\{ -\sum_{j=1}^{N_o} \int\limits_0^T \psi_{x_\gamma}(\eta^i,t) k_i^j \left[u\left(\xi^j,t\right) - z_i^j\right] dt +$$

$$+ 4\sum_{j=1}^{N_o} \mathcal{R}_{N_c+(i-1)N_o+j}\left(\xi_\gamma^j - \eta_\gamma^i\right) g_{N_c+(i-1)N_o+j}^+(\cdot, y)$$

$$+ 2\varepsilon\left(\eta_\gamma^i - \hat{\eta}_\gamma^i\right)\Bigg\} \rho_\Theta(\theta)\rho_\Phi(\varphi)\, d\theta\, d\varphi,$$

$i = 1,2,\ldots,N_c$, $j = 1,2,\ldots,N_o$, $\gamma = 1,2$. Here function $\psi(x,t) = \psi(x,t;y,\varphi,\theta)$ is the solution of following adjoint boundary-value problem:

$$\psi_t(x,t) = -a^2\,\mathrm{div}(\mathrm{grad}\,\psi(x,t)) + \lambda_0\psi(x,t) -$$

$$-\sum_{j=1}^{N_o}\sum_{i=1}^{N_c} k_i^j\left(\psi\left(\eta^i,t\right) + 2\mathcal{R}_i g_i^+(t;y)\,\mathrm{sgn}(g_i^0(t;y))\right)\delta(x - \xi^j)\,,$$

$$x \in \Omega, \quad t \in [0,T),$$
$$\psi(x,T) = -2\mu(x)[u(x,T;y) - U(x)], \quad x \in \Omega,$$
$$\frac{\partial\psi(x,t)}{\partial \mathrm{n}} = \lambda\psi(x,t), \quad x \in \Gamma, \quad [0,T).$$

In the **third chapter**, high-order approximation methods are proposed and solution formulas are obtained for the numerical solution of a system of non-autonomous linear ordinary differential equations given by non-local (non-separated) intermediate conditions. For Dirac's two-dimensional δ-function, a method of approximation with a smooth function everywhere has been proposed.

In **paragraph 3.1**, the system of linear ordinary differential equations with boundary and intermediate conditions $L-1$ with non-separated conditions consisting of a number of points is considered:

$$\dot{x}(t) = A(t)x(t) + B(t), t \in [t_0, T], \tag{51}$$

$$\sum_{i=0}^{L} C^i x(\bar{t}_i) = d, \tag{52}$$

here, $x(t) \in \mathrm{R}^n$ is n dimensional vector-function; $A(t)$ is the continuous n-dimensional square matrix-function, $A(t) \neq \text{const}$, $t \in [t_0, T]$, $B(t)$ is continuous n dimensional vector-function, C^i given verilmiş n-dimensional square matrices, $i = 0,1, \dots, L$; d is a given n dimensional vector; $\bar{t}_i \in [t_0, T]$ moments of time and t_0, T given, where, $t_0 = \bar{t}_0 < \bar{t}_1 < \dots < \bar{t}_L = T$.

In order to increase the accuracy of the numerical solution of the system (51) in the conditions (52) it is proposed to use multipoint approximation schemes for the derivative $\dot{x}(t)$ in a given grid:

$$\omega = \left\{ \begin{matrix} \tau_j \in [t_0,T]: \ \ \tau_j = jh, \ \ j = 0,1, \dots, N, \ \ \tau_0 = t_0, \ \ \tau_N = T, \\ h = (T - t_0)/N \end{matrix} \right\}.$$

Assume that the points, $\bar{t}_i$, $i = 0,1, \dots, L$ belong to a given ω şəbəkə grid area, i.e.

$$\bar{t}_i = \tau_{s_i}, \qquad i = 0,1, \dots, L,$$

here, s_i is the sequence number in the grid i-th point $\bar{t}_i$.

Obviously, the known k-step approximation formulas for $\dot{x}(t)$ can be used:

$$\dot{x}(t)|_{t=\tau_j} = \sum_{\gamma=k_1}^{k_2} \alpha_\gamma x^{j+\gamma} + O(h^m). \qquad (53)$$

Here $k_1 + k_2 = k,\ k_1 \geq 0,\ k_2 \geq 0,\ x^j = x(\tau_j) \in \mathrm{R}^n$, (53) a is the approximation accuracy from the order m determined by the approximation scheme.

For the derivative $\dot{x}(t)$, $O(h^m)$ has any k steps of precision from the order:

$$x^j = \sum_{\gamma=k_1^j}^{k_2^j} \alpha_\nu^j x^{j+\gamma} + \beta^j, \qquad (54)$$

if we use the approximation scheme, we can obtain a system of discrete linear equations with k steps defined by non-local intermediate conditions not separated as follows:

$$\sum_{i=0}^{L} C^i x^{s_i} = d. \qquad (55)$$

In relations (54), the coefficients α_γ^j are determined by the coefficients of the differential equations (51) and the approximation of the difference scheme (53). The values k_1^j and k_2^j satisfy the following conditions:

$$k_1^j + k_2^j = k,\ \ j = 0,1, \dots ,N,$$

$j = N - k + 1, N - k + 2, \dots ,N$ for k_1^j və k_2^j get different values ((54) approximation scheme), but $j = 0,1, \dots ,N - k$ for i.e., ω for the nodes of the internal node of the grid domain k_1^j və k_2^j have the same values and, as a rule, the scheme does not change and is determined by the following equations:

$$x^j = \sum_{\gamma=1}^{k} \alpha_\gamma^j x^{j+\gamma} + \beta^j,\ \ j = 0,1,\dots,N - k. \qquad (56)$$

Teorem 6. The following

$$\tilde{C}_\gamma^1 = C^i \alpha_\gamma^0,\ \ \gamma = 1,2,\dots,k,$$

$$\tilde{C}_\gamma^{j+1} = \tilde{C}_1^j \alpha_\gamma^j + \tilde{C}_{\gamma+1}^j,\ \ \gamma = 1,2,\dots,k - 1,\ \ \tilde{C}_k^{j+1} = \tilde{C}_1^j \alpha_k^j,$$

$$j = s_0 + 1, \ldots, s_1 - k - 1,$$
$$\tilde{C}_k^{j+1} = \tilde{C}_1^j \alpha_k^j + C^2, \quad j = s_1 - k,$$
$$\tilde{d}^1 = d - C^0 \beta^0, \;\; \tilde{d}^{j+1} = \tilde{d}^j - \tilde{C}_1^j \beta^j, \quad j = s_0 + 1, \ldots, s_1 - k,$$
the matrices $\tilde{C}_\gamma^j$, $\gamma = 1, \ldots, k$, $j = s_0, \ldots, s_1 - k$ and vectors $\tilde{d}^j$, $j = s_0, \ldots, s_1 - k$ obtained from recurrent relations (53) with respect to the discrete system (52) are the coefficients of the conditions.

In **paragraph 3.2** analysis of impulse influences in the lumped parameter systems, and in **paragraph 3.3** analyses the methods of approximation of the lumped (point-wise) sources in the distributed parameter systems was studied.

A "sinusoid-like" function was proposed for the approximation of Dirac's one-dimensional δ-function:
$$\tilde{\delta}_\varepsilon(x;\xi) = \begin{cases} 0, & |x - \xi| > \varepsilon, \\ \dfrac{1}{2\varepsilon}\left[1 + \sin\left(\dfrac{2x - 2\xi + \varepsilon}{2\varepsilon}\pi\right)\right], & |x - \xi| \le \varepsilon. \end{cases}$$
and the proposed approximation scheme was analyzed.

In **paragraph 3.3**, Dirac's two-dimensional δ-function is approximated by the following "sinusoid-like" $\tilde{\delta}(x;\eta)$ function, which is smooth everywhere:
$$\tilde{\delta}_\varepsilon(x;\eta) =$$
$$= \begin{cases} 0, & |x_1 - \eta_1| \ge \varepsilon_1 \text{ və ya } |x_2 - \eta_2| \ge \varepsilon_2, \\ \displaystyle\prod_{i=1}^{2} \frac{1}{2\varepsilon_i}\left[1 + \sin\left(\frac{2x_i - 2\eta_i + \varepsilon_i}{2\varepsilon_i}\pi\right)\right], & |x_1 - \eta_1| < \varepsilon_1 \;\text{ və }\; |x_2 - \eta_2| \le \varepsilon_2. \end{cases}$$

The fourth chapter provides information on the proposed algorithms designed to perform computer experiments. Block diagrams describing the working principles of algorithms are shown. Software developed in C/C ++ programming language. The software includes modules describing various one-dimensional minimization methods and gradient methods, as well as modules for solving different loaded partila differential equations, through which one-dimensional or two-dimensional differential equations can be solved with high accuracy and stabl conditions with the help of grid, variable direction and high-precision methods.

In the **Appendix**, the numerical solution of the initial-boundary value problems described by the parabolic and hyperbolic type partial differential equations, gradient methods for finding the optimal solution of control problems, etc. The source code of the software modules developed by the author is given.

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MAIN RESULTS OF WORK

In the dissertation, the following results were obtained.

1) The synthesis of the points and power of the lumped control impacts in the process of calming the oscillations of the membrane with feedback, as well as the coordinates of the measurement points was considered and the gradient formulas of the objective function for these parameters were obtained.
2) The problem of synthesis of optimal feedback parameters of boundary control in distributed parameter systems has been studied and gradient formulas of the objective function for them have been obtained.
3) In the process of heating the plate with point-wise heat sources, the synthesis of control, the optimal control and measurement location points, the optimization of feedback parameters were studied and the gradient of the objective function were obtained.
4) Analytical formulas of the gradient of the objective functional according to the feedback parameters, coordinates of the points of measurement and control sources were obtained in the considered problems, which the problems were solved numerically using the first-order optimization methods.
5) Schemes of using high-order approximation methods of equations for numerical solution of non-autonomous linear ordinary differential equations system with non-local intermediate conditions are proposed.
6) Software for computer experiments has been developed based on the proposed methods and algorithms for the problems under consideration. Numerical experiments were executed on test problems using the software, and their results were given.

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