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ABSTRACT

of the dissertation for the degree of Doctor of Science

NUMERICAL SOLUTION TO SOME CLASSES OF PARAMETRIC IDENTIFICATION AND OPTIMAL CONTROL PROBLEMS

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GENERAL DESCRIPTION OF THE WORK

Relevance of the topic. It is known that the study of a lot of dynamic objects and processes is carried out in stages, among which there are two main ones: 1) the construction of an adequate mathematical model; 2) optimization and optimal control of the studied object.

One of the first systematic expositions of diverse algorithms and identification methods is prof. Eichhoff's work. Among the most significant works dedicated to the identification of linear dynamical systems, one can mention the works of D. Gropp, E.P. Sage and J.L. Melsa, L. Lyung, Ya.Z. Tsypkin, N.S. Raibman, S.E. Steinberg, etc.

The inverse problems of mathematical physics are closely related to the problems of parametric identification of mathematical models. Their most active study was started by such scientists as A.N. Tikhonov, M.M. Lavrentiev, A.A. Samarsky and others. Over the past two decades, these problems have been given great attention by a lot of scientists such as P.N. Vabishchevich, V.K. Ivanov, Yu.M. Kulibanov, S.I. Kabanikhin, A.L. Karchevsky, etc., including domestic scientists such as A. Iskenderov, R. Tagiev, V.M. Abdullaev, A.B. Ragimov, A.Ya. Akhundov, Y.T. Megraliev, etc.

The theory of control of nonlinear systems is an extremely important and actively developing field of science. In the general case, a lot of physical systems, and technical systems in particular, are, as a rule, non-linear and are characterized by multidimensionality, non-stationarity, large dimension, and uncertainty of the underlying mathematical models. The control synthesis laws for such systems is often associated with significant difficulties, both theoretical and computational in nature. In solving these problems, an important role is played by the theory of optimal control of both objects with lumped (OLP) and distributed parameters (ODP). Despite its already long history of development, the most serious results were obtained only in the 50-60s of the last century. A great role in this development was played by such scientists as R.E. Bellman, L.S. Pontryagin, R.F. Gabasov, F.M. Kirillova, etc. Considerable contribution was made by domestic scientists, such as G. Akhmedov, A. Iskenderov, S. Hasanov, M. Mardanov, K. Mansimov, M. Yagubov, K. Aida-zade, R. Tagiev, G. Guliev, T. Melikov, I. Aliyev, etc. But the biggest problem in the theory and practice of control is the problem of the synthesis of control actions under various types of feedback from the object.

The problem of synthesis of control with feedback has been sufficiently studied for linear systems in the case when the state vector is accessible to measurement. In practice, only the output vector, functionally associated with the state vector, is available for direct measurement. This fact obviously leads to the need to solve the problem of synthesis of stabilizing control with feedback on the output. Despite the naturalness of such a statement, this problem remains not fully studied even for linear systems with static feedback. A review of some results on this topic can be found in the works of B.T. Polyak and P.S. Scherbakov, V.L. Syrmos, C.T. Abdallah, P. Dorato, and K. Grigoriadis. Recent results on discrete systems are presented in the works of G. Garcia, B. Pradin, S. Tarbouriech, and F. Zeng. Important results in this direction in the republic were obtained by F.A. Aliyev and his students. By now, a number of necessary and sufficient stabilization conditions have been obtained using static output feedback. (D. Youla, V. Kucera, T. Iwasaki, R. Skelton, etc.). At the same time, control practice requires the solution of such problems, including under conditions of uncertainty in the parameters of the object (robust stabilization). In accordance with this, it seems important to develop methods and algorithms for solving the stabilization and robust output stabilization problems based on sufficient conditions and constructive heuristic procedures.

In recent decades, a number of effective methods for the synthesis of control of nonlinear objects have been developed. Significant results in the development of control synthesis methods for nonlinear objects were obtained in the works of F.A. Aliev and V.B. Larin, A.A. Krasovsky, Y.Z. Tsypkin, V.A. Yakubovich, A.I. Egorov, T.K. Sirazetdinov, A.A. Bobtsov, I.V. Miroshnik, V.O. Nikiforov and A.L. Fradkov, D.J. Hill and P. Moylan, A. Isidori, I. Kanellakopoulos and M. Karstic, N.K. Khalil, P.V. Kokotovic, R. Marino and P. Tomei, K.S. Narendra, S.S. Sastry, E.D. Sontag, etc.

Feedback control problems are complicated in the case of objects described by essentially nonlinear systems and when it is impossible to organize continuous observation of the state of the object. The linearization methods of nonlinear systems lead to large approximation errors, and as a result, the corresponding linearized control and regulation systems are not sufficiently adequate to the ongoing processes. On the other hand, the theoretical results and methods developed for linear systems are practically not applicable to nonlinear systems, since they lead to complex computational problems, the solution of which is required for some approaches to be carried out at a pace with the process (in real time).

The dissertation is dedicated to the study of the above-mentioned problems of parametric identification of systems with both lumped and distributed parameters, as well as of the problems of synthesis of control actions for these systems. To solve these problems, a unified approach is used, which lies in introducing the concept of "zonal parameters", both for identification and control actions.

Purpose of work:

- Development and mathematical justification of methods for solving optimization and parametric identification problems for OLP and ODP, with discontinuous right-hand sides, illustration of their application in solving practical problems.
- Software development in the form of packages of applied optimization programs, as well as the development of a control system in automatic and interactive (dialog) modes using modern information and computer technologies, and parallel computing.

The scientific novelty of the dissertation work is as follows:

- We proposed and justified a numerical method for solving the problems of parametric identification of dynamic, in the general case, nonlinear OLP described by systems of nonlinear differential equations with ordinary derivatives.

- We proposed an approach to solving a class of inverse problems on determining switching surfaces depending on the state of a dynamic process described by a discontinuous system of ordinary differential equations.
- We proposed an approach to numerical solution to the problem of synthesis of optimal control of objects described by systems of nonlinear ordinary differential equations under various types of feedback on input/output and on different classes of zonal control functions given inaccurate information on the values of the initial conditions and object parameters.
- For decision-making systems of controlling dynamic processes, we proposed an approach that consists in combining the stages of parametric identification of a mathematical model and of optimization of the regimes.
- We solved the problems of identifying nonlinear coefficients of mathematical models and control synthesis with respect to specific ODPs using the proposed approaches.
- We proposed a numerical method for solving the problem of identifying the hydraulic resistance coefficient, which depends on the regimes of movement of hydrocarbon feedstock in a linear section of the pipeline, described by a system of hyperbolic equations.
- For the ODP, by the example of controlling the heating process by a heat exchanger with a delay in boundary conditions, we proposed an approach to the synthesis of control actions based on continuous observation of the phase state of the object at its certain points.
- Using the example of control problems for the processes of heating the rod and plate, we proposed an approach to the synthesis of control of lumped sources for ODP based on continuous observation of the phase state at certain points of the object.
- We developed mathematical packages and software for solving complex optimization problems based on interactive and automatic control of the unconditional optimization software package using parallel computations on modern computer systems with a

multiprocessor/multicore architecture. The developed systems are equipped with an extensive library of optimization algorithms.

General research methodology. In the dissertation we used mathematical methods for modeling non-stationary processes; numerical methods for solving differential equations, optimal control, finite-dimensional optimization; modern information technology and programming tools.

Theoretical and practical value. The scientific and practical value of the problems considered in the dissertation is that the approach used to mathematical modeling of the processes under study and their subsequent optimal control, as well as the corresponding computer codes, can be used to solve a very wide range of problems from such fields of science and technology as design technology, construction and use of robots, production systems, technologies for converting power (energy), planning medical treatment, interpretation of remote sensing data, etc. It should be noted that the problems of identification and optimal control, considered in the dissertation, as well as the corresponding algorithms are published in the scientific press, and are, therefore, publicly available for use. The scientific validity and reliability of the obtained results is confirmed by the theoretical rigor of the formulas, equations and relationships used in the algorithms, by various testing of algorithms and corresponding computer codes, including verification using special test systems (including those developed by the author of the dissertation), as well as by comparison with the results of similar independent calculations.

Work approbation. The main results of the work were reported at the following international conferences: International Conference on “Control and Optimization with Industrial Applications” COIA-2005, -2008, -2013, -2015; International Conference “Problems of Cybernetics and Informatics” PCI-2006, -2008, -2010, -2012; 24th Mini Euro Conference “Continuous Optimization and Information-Based Technologies in the Financial Sector, (MEC EurOPT), 2010 (Turkey, Izmir); International Russian-Bulgarian Symposium “Equations of a mixed type and related problems of analysis and computer science”, Nalchik, Russia, 2010; International Russian-

Kazakh Symposium "Equations of a mixed type and related problems of analysis and computer science", Nalchik, Russia, 2011 and 2014; International Russian-Abkhazian Symposium "Equations of the mixed type and related problems of analysis and computer science", Nalchik, Russia, 2009; International Russian-Uzbek Symposium "Equations of a mixed type and related problems of analysis and computer science", Nalchik, Russia, 2012; IV International Conference "Nonlocal boundary-value problems and related problems of mathematical biology, computer science and physics", Nalchik-Treskol, Russia, 2013; International conference "Actual problems of modern mathematics, computer science and mechanics-II", Kazakhstan, Almaty, 2011; the 4th Congress of the Turkic World Mathematical Society (TWMS), Baku, 2011; International Conference «Optimization Methods and Applications» (OPTIMA), Costa Da Caparica, Portugal, 2012; International Conference «Optimization Methods and Applications» (OPTIMA), Petrovac, Montenegro, 2011, 2014; VI International Conference (MPMO-2017), Russia, Ulan-Ude, 2017; International scientific-practical conference "Innovative technologies in the oil and gas industry", Russia Stavropol, 2015; International Conference “Applied Mathematics and Fundamental Computer Science”, Omsk, Russia, 2016, 2017; International Scientific Conference "Informatics and Applied Mathematics" dedicated to the 25th anniversary of Independence of the Republic of Kazakhstan and the 25th anniversary of the Institute of Information and Computational Technologies, Kazakhstan, Almaty, 2016; International conference "Actual problems of mathematics and mechanics" dedicated to the 80th anniversary of Honored Scientist, Y.J. Mamedov, 2010, Baku; International conference "Oil-gas, oil refining and petrochemicals" dedicated to the 90th anniversary of the ASOIU, 2010; International conference "Actual problems of mathematics and computer science" dedicated to the 90th anniversary of the birth of Heydar Aliyev, Baku, 2013; International Conference "Non-Newtonian Systems in the Oil and Gas Industry" dedicated to the 85th anniversary of Academician A.Kh. Mirzadzhanzade, Baku, 2013; XI International Chetaev scientific conference "Analytical

Mechanics, Stability and Control", Kazan, 2017; International conference "Actual problems of applied mathematics and physics", Elbrus region, Kabardino-Balkaria, Russian Federation, 2017.

The results of the research work were also reported at scientific seminars of the following scientific organizations: the Department of General and Applied Mathematics of the ASOIU; the Laboratory of Decision Making Methods in Deterministic Systems of the Institute of Control Systems of the National Academy of Sciences of Azerbaijan; the Institute of Mechanics and Mathematics of the National Academy of Sciences of Azerbaijan; the Research Institute of Applied Mathematics Institute at BSU; the faculties of computer sciences of Yashar and Dokkuz Eilul Universities (Izmir, Turkey); the Department of Numerical Optimization Methods of the Institute of Cybernetics of the National Academy of Sciences of Ukraine; as well as at many national and international conferences.

Publications. On the topic of the dissertation, 71 scientific papers have been published; 26 of which are full papers, 18 of which are published in foreign countries, 18 of which are included in the Scopus database; 11 of which are included in the international Web of Science™ Core Collection database by Thomson Reuters; 29 short papers are published in international conferences.

Volume and structure of work. The dissertation work consists of an introduction, six chapters, a conclusion, an appendix, and a bibliography containing 208 titles. The total volume of the dissertation is 329 pages of typewritten text, the main volume is 238 pages, including 15 tables and 15 figures. The title page contains 441 characters, the table of contents - 3382 characters, the introduction - 76411 characters, the content of the dissertation - 315508 characters (Chapter 1 - 35676 characters, Chapter 2 - 37506 characters, Chapter 3 - 75314 characters, Chapter 4 - 41346 characters, Chapter 5 - 71752 sign, chapter 6 - 53914 characters), conclusions - 2597 characters, references - 32869 characters, totally - 431208 characters.

THE CONTENT OF THE WORK

In the **introduction**, the relevance of the topic of the dissertation is substantiated, its main purpose is formulated, and a brief review of the work is given.

In the **first chapter** of the dissertation, consisting of 3 paragraphs, we study the problem of identifying the coefficients of a mathematical model of an object depending on its phase state.

The dynamic objects studied in the first chapter are described in the general case by a nonlinear system of differential equations:

$$\dot{x}(t) = f\left(x(t), K\big(x(t)\big)\right), \ \ t \in (0, T], \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the continuous and almost everywhere continuously differentiable vector-function of the phase state; $K(x) \in \mathbb{R}^r$ is the identifiable continuous and almost everywhere continuously differentiable vector-function that determines the coefficients of the mathematical model; the known vector function $f(x, K)$ is continuously differentiable with respect to all its arguments.

In order to identify the coefficients of the mathematical model of the process (1), it is assumed that there are N independent observations of the dynamics of the state of the object under various initial states:

$$x^i(0) = x_0^i, \ \ i = 1,2, \dots, N. \tag{2}$$

The results of observations can be any components or the entire state vector at individual points in time

$$x\big(t_{ij}; x_0^i\big) = x^{ij}, \ \ t_{ij} \in (0, T], \ \ j = 1,2, \dots, M_i, i = 1,2, \dots, N, \tag{3}$$

in particular, at a finite point in time T:

$$x\big(T; x_0^i\big) = x_T^i, \ \ i = 1,2, \dots, N, \tag{4}$$

where M_i is the number of times at which observations were made on the state of the object with the initial condition x_0^i. The state of the object can be observed under various initial conditions at certain time intervals:

$$x\big(t; x_0^i\big) = y^{ij}(t), \ t \in \left[\tau_{ij-1}, \tau_{ij}\right] \in [0, T], \tau_{ij-1} < \tau_{ij}, \\ j = 1,2, \dots, M_i; i = 1,2, \dots, N, \tag{5}$$

where M_i is the number of time intervals in which observations were made on the state of the object with the initial state x_0^i. Observations can also be of a mixed type, i.e. both point (3) or (4), and interval (5).

The problem under consideration consists in determining (identifying) unknown coefficients $K(x)$ of system (1) based on the results of observations of the form (2), (3), (4), or (5).

The quality of identification is evaluated using the least squares criterion, and for each type of observation (3) – (5), the specific form of the criterion is different. In the case of final observations (4), the identification quality criterion will take the following form:

$$\mathrm{J}(K(x)) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{J}^i\left(x\left(T; x_0^i, K(x)\right)\right) + \\ +\varepsilon\left\|K(x) - \widehat{K}(x)\right\|_{L_2^r}^2 \to \min_{K(x)}, \qquad (6)$$

$$\mathrm{J}^i\left(x\left(T; x_0^i, K(x)\right)\right) = \left\|x\left(T; x_0^i, K(x)\right) - x_T^i\right\|_{\mathbb{R}^n}^2,$$

where $x(t) = x\left(T; x_0^i, K(x)\right)$ is the solution to problem (1) under any initial condition x_0^i and the coefficients determined by the vector function $K(x) \in \mathbb{R}^r$, $\varepsilon > 0$ and $\widehat{K}(x)$ are the regularization parameters.

To recover the unknown coefficients of the system of differential equations (1), we propose an approach based on which the entire set of possible phase states is partitioned into a finite number of subsets, in each of which the coefficients are searched for on a parametrically specified class of functions depending on the phase state determined using some basis functions. In this case, the initial task is reduced to the determination of the constant parameters involved in the representation of the coefficients.

Denote by $X \subseteq \mathbb{R}^n$ the set of all possible phase states of the object $x(t)$ under all admissible values of the initial conditions and values of the coefficients $K(x)$. Let X be portioned into a given finite number L of simply connected subsets (zones) $X^k \subset X$. Phase space zones

$$X^\nu = \{x \in \mathbb{R}^n : g^{\nu-1}(x) > 0, g^\nu(x) \le 0\},\ \nu = 2,3,\dots,L-1,$$

$$X^1 = \{x \in \mathbb{R}^n: g^1(x) \le 0\},\ \ X^L = \{x \in \mathbb{R}^n: g^{L-1}(x) > 0\},$$

are defined by their boundaries using the functions $g(x) = (g^1(x), g^2(x), \dots, g^{L-1}(x))$ continuously differentiable and defined almost everywhere.

The identifiable coefficients $K(x) = (k_1(x), k_2(x), \dots, k_r(x))$, in each of the zones $X^\nu, \nu = 1,2,\dots,L$, are defined in the form of the following representation:

$$\begin{gathered} K(x) = K^\nu(x) = (k_1^\nu(x), k_2^\nu(x), \dots, k_r^\nu(x)) \in \mathbb{R}^r, \\ k_s^\nu(x) = \sum_{i=1}^{m} p_{si}^\nu \phi_i(x), \qquad p_{si}^\nu = \text{const}, \\ s = 1,2,\dots,r,\ \ i = 1,2,\dots,m; \\ x \in X^\nu, \nu = 1,2,\dots,L, t \in (0,T], \end{gathered} \tag{7}$$

where $\phi_i(x)$, $i = 1,2,\dots,m$, are given continuously differentiable linearly independent basis functions; p_{si}^ν are yet unknown constant parameters defining identifiable functions. As a rule, the coefficients $K(x)$, in real problems, must satisfy the constraints based on technical and technological considerations; and therefore the parameters $p = (p^1, p^2, \dots, p^L)$, $p^\nu = (p_{11}^\nu, p_{12}^\nu, \dots, p_{1m}^\nu, \dots, p_{rm}^\nu)$, $\nu = 1,2,\dots,L$, must also satisfy certain relevant restrictions. We denote the sets of permissible values of the zonal parameters p^ν by $P^\nu \subset \mathbb{R}^{r\times m}$, $\nu = 1,2,\dots,L$, which are assumed to be closed and bounded, $P = P^1 \times P^2 \times \dots \times P^L$.

In this case, the solution of the system of differential equations (1), which determines the current state of the process $x(t)$, will depend on the initial state x_0 and zonal values of the parameter vector p, т.е. $x(t) = x(t; x_0, p)$; moreover, inside each zone, $x(t)$ is continuously differentiable, and at the points of transition from one zone to another – it is continuous.

Thus, the initial problem of determining the coefficients $K(x)$ (1) – (6) is replaced by the problem of determining constant parameters $p \in \mathbb{R}^{L\times r\times m}$, by which the desired coefficients are approximated.

The identification criterion (6), taking into account the representation (7), will take the form:

$$\mathrm{J}(p) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{J}^i\left(\mathrm{x}\left(T; \mathrm{x}_0^i, p\right)\right) + \varepsilon\|p - \hat{p}\|^2_{\mathbb{R}^{L\times r\times m}} \to \min_{p\in P}, \qquad (8)$$
$$\mathrm{J}^i\left(\mathrm{x}\left(T; \mathrm{x}_0^i, p\right)\right) = \left\|\mathrm{x}\left(T; \mathrm{x}_0^i, p\right) - \mathrm{x}_T^i\right\|^2_{\mathbb{R}^n},$$

where $\mathrm{x}(t) = \mathrm{x}\left(t; \mathrm{x}_0^i, p\right)$ is the solution to the Cauchy problem (1) for a given admissible vector of parameters p and the initial state x_0^i, taking into account the representation (7); $\hat{p}$ is the regularization parameter corresponding to the function $\widehat{K}(\mathrm{x})$ from (6).

Problem (1), (2), and (8) can be attributed to the problem of parametric optimal control. At the same time, this problem, due to the fact that a finite-dimensional vector of parameters p is being optimized, also fits into the framework of finite-dimensional optimization problems. To solve it, one can use well-known effective numerical optimization methods, in particular, of the first-order, and ready-made standard software. To this end, as is known, it is necessary to obtain formulas for calculating the components of the gradient of the objective functional (8) with respect to the components of the vector $p - \nabla_p \mathrm{J}(p)$.

For an arbitrary number of zones of the phase space, in the general case, we obtained formulas for the gradient of the target functional

$$\frac{\mathrm{d}\,\mathrm{J}(p)}{\mathrm{d}\,p_{kj}^l} = \frac{1}{N}\sum_{i=1}^{N} \frac{\mathrm{d}}{\mathrm{d}\,p_{kj}^l}\mathrm{J}^i\left(\mathrm{x}\left(T; \mathrm{x}_0^i, p\right)\right) + 2\varepsilon\left(p_{kj}^l - \hat{p}_{kj}^l\right),$$

$$\frac{\mathrm{d}}{\mathrm{d}\,p_{kj}^l}\mathrm{J}^i\left(\mathrm{x}\left(T; \mathrm{x}_0^i, p\right)\right) = \int_{\Pi_l(\mathrm{x}_0^i, p)} \left[\psi^*\left(t; \mathrm{x}_0^i, p\right) \cdot \right.$$
$$\left. \cdot \frac{\partial f\left(\mathrm{x}\left(t; \mathrm{x}_0^i, p\right), K^l\right)}{\partial K^l} \cdot \frac{\partial K^l}{\partial p_{kj}^l}\right] \mathrm{d}t,$$

where $\Pi_l\left(\mathrm{x}_0^i, p\right)$, $l = 1,2,\dots,L$, $i = 1,2,\dots,N$ is the period of time during which the trajectory with the initial condition x_0^i and the value of the parameters p was in the zone X^l; $\psi\left(t; \mathrm{x}_0^i, p\right)$ is the solution to the following adjoint system:

$$\dot{\psi}^*(t; x_0^i, p) = -\psi^*(t; x_0^i, p) \cdot \frac{\partial f(x(t; x_0^i, p), K^l)}{\partial x}, \ t \in \Pi_l(x_0^i, p),$$

$$\psi(T; x_0^i, p) = \frac{\partial J^i\left(x(T; x_0^i, p)\right)}{\partial x},$$

satisfying the following condition of a jump at the time when the trajectory of system (1) hits the boundary between zones:

$$\psi(\bar{t}_l - 0) = \psi(\bar{t}_l + 0) - \frac{\partial g\left(x(\bar{t}_l)\right)}{\partial x} \cdot \gamma,$$

$$\gamma = \frac{\psi^*(\bar{t}_l + 0) \cdot [f(x(\bar{t}_l), K^l) - f(x(\bar{t}_l), K^{l+1})]}{\frac{\partial g^*\left(x(\bar{t}_l)\right)}{\partial x} \cdot f(x(\bar{t}_l), K^l)}$$

$$l = 1,2, \dots, L-1.$$

At the end of the first chapter, we present the results of numerical experiments by the example of solving several test problems using software developed by the author.

In the **second chapter** of the dissertation, consisting of 3 paragraphs, we study the problem of parametric identification of discontinuous dynamic OLPs. We consider a class of inverse problems for dynamic processes described by discontinuous (variable structure, compound, stepwise, etc.) systems of ordinary differential equations, the form of which varies depending on whether the process state belongs to one or another subregion of the state space. These problems have been considered by a lot of authors. In contrast to previous studies, we identify the switching surfaces themselves. We obtained formulas for the components of the gradient of the objective functional with respect to identifiable parameters.

Suppose that the dynamics of the studied object is described by a system of nonlinear differential equations with a variable structure of the form:

$$\dot{x}(t) = f^l\left(x(t), p^l(t)\right), x(t) \in X^l(t), \ t \in (0, T], \qquad (9)$$

$$l = 1,2, \dots, L,$$

where $x(t) \in \mathbb{R}^n$ is the vector that determines the state of the process; $p^l(t) \in \mathbb{R}^{r_l}$ are parameter values when the state of the process belongs to $X^l(t)$ – subregion (zone) of the phase space of all possible states of the process X, i.e. $X^l(t) \subset X \subseteq \mathbb{R}^n$, $l = 1,2, \dots, L$. The vector-functions $f^l(.,.)$, $l = 1,2, \dots, L$, given to within the functional parameters $p^l = p^l(t)$, are continuously differentiable with respect to all their arguments.

Phase space zones

$$X^l(t) = \{x(t) \in \mathbb{R}^n : g^{l-1}(x,t) > 0, g^l(x,t) \le 0\},$$
$$l = 2,3, \dots, L-1, \tag{10}$$
$$X^1(t) = \{x(t) \in \mathbb{R}^n : g^1(x,t) \le 0\},$$
$$X^L(t) = \{x(t) \in \mathbb{R}^n : g^{L-1}(x,t) > 0\},$$

simply connected and determined by their boundaries using identifiable almost everywhere twice continuously differentiable functions $g(x,t) = \left(g^1(x,t), g^2(x,t), \dots, g^{L-1}(x,t)\right)$, where

$$\operatorname{int} X^{l_1}(t) \cap \operatorname{int} X^{l_2}(t) = \emptyset, \quad l_1 \ne l_2, \quad l_1, l_2 = 1,2, \dots, L,$$

$$\bigcup_{l=1}^{L} X^l(t) = X.$$

The vector-function $x(t)$, the solution to system (9), is everywhere continuously differentiable except at the points of time $\bar{t}_l$ when the trajectory of the system hits the switching surface, i.e. $g^l\left(x(\bar{t}_l), \bar{t}_l\right) = 0$, $l = 1,2, \dots, L-1$, at which $x(t)$ is continuous. We introduce the notation

$$p(t) = \left(p^1(t), p^2(t), \dots, p^L(t)\right) =$$
$$= \left(p_1^1(t), p_2^1(t), \dots, p_{r_1}^1(t), p_1^2(t), \dots, p_{r_L}^L(t)\right) \in \mathbb{R}^r,$$
$$r = \sum_{l=1}^{L} r_l.$$

It is assumed that, in order to identify unknown parameters, N independent observations were made of the dynamics of the object under various initial states:

$$x^i(0) = x_0^i,\ i = 1,2,\dots,N. \tag{11}$$

In this case, the current state of the process $x(t)$ depends on its initial state x_0, on the function $g(x,t)$ that defines the region $X^l(t)$, $l = 1,2,\dots,L$, and the corresponding parameter values $p(t)$, i.e. $x(t) = x(t; x_0, p, g)$. Observations can be carried out for any components or for the entire state vector of an object at separate intervals or time instants, in particular, at a finite moment in time T:

$$x^i(T; x_0^i, p, g) = x_T^i,\ \ i = 1,2,\dots,N, \tag{12}$$

where N is the number of time points at which observations were made in the i[th] experiment over the state of the object with the initial condition x_0^i.

We considered the most frequently encountered case in practice when the identified parameters by zones are piecewise constant functions, namely

$$\begin{gathered} p^l\,(t) = p^l = \text{const},\ p^l \in \mathbb{R}^{r_l}, x(t) \in X^l, \\ l = 1,2,\dots,L;\ \ t \in (0,T]. \end{gathered} \tag{13}$$

The considered problem is to determine the $(L-1)$-dimensional vector function $g(x,t)$ and the finite-dimensional vector $p \in \mathbb{R}^r$. For each type of observation, an appropriate identification quality criterion should be selected. In the case of observations of the form (12), the mean square quality criterion can be used:

$$\begin{gathered} J(p,g) = \frac{1}{N}\sum_{i=1}^{N} J_i\big(x^i(T; x_0^i, p, g), p, g\big), \\ J_i\big(x^i(\cdot), p, g\big) = \left\| x^i(\cdot) - x_T^i \right\|_{\mathbb{R}^n}^2. \end{gathered} \tag{14}$$

The identification problem is reduced to the parametric optimal control problem, which consists in minimizing functional (14) under conditions (9) – (12).

To determine the functions $g^l(x,t)$, $l = 1,2,\dots,L-1$, we propose to parameterize them using any finite system of linearly independent continuously differentiable functions $\{\phi^i(x,t)\}$, $i = 1,2,\dots,\overline{k}$, using the following representations for the functions $g^l(x,t)$, $l = 1,2,\dots,L-1$:

$$g^l(x,t) = g^l(x,t;\alpha^l) = \sum_{i=1}^{k_l} \alpha_i^l \, \phi^i(x,t)\,,\ l = 1,2,\dots,L-1$$

$$\alpha^l \in \mathbb{R}^{k_l},\ \ k = \sum_{l=1}^{L-1} k_l\,,\ \ \overline{k} = \max_{1\le l\le L-1} k_l,$$

$$\alpha = \left(\alpha_1^1, \alpha_2^1, \dots, \alpha_{k_1}^1, \alpha_1^2, \dots, \alpha_{k_{L-1}}^{L-1}\right) \in \mathbb{R}^k.$$

In this case, the problem of determining the function $g(x,t)$ is replaced by the problem of identifying the vector α.

Thus, (9) – (14) is a problem of parametric identification with respect to a finite-dimensional vector $z = (p,\alpha) \in \mathbb{R}^{r+k}$. We obtained formulas for the components of the gradient of the objective functional $\nabla J(z) = \left(\nabla_p J(z), \nabla_\alpha J(z)\right)$, which allow us to formulate the necessary first-order optimality conditions, as well as to use effective first-order numerical methods and ready-made software tools to solve the identification problem.

Theorem 2.1. For the optimality of the vector α^* in problem (9) – (14), it is necessary that the condition

$$\frac{\partial J(p,g)}{\partial \alpha_s^l} = \frac{1}{N}\sum_{i=1}^{N} \frac{\partial}{\partial \alpha_s^l} J_i\left(x^i\left(T; x_0^i, p, g\right), p, g\right) = 0,$$

$$\frac{\partial}{\partial \alpha_s^l} J_i\left(x^i\left(T; x_0^i, p, g\right), p, g\right) = \sigma_i^l \cdot \phi^s\left(x\left(\overline{t}_{l,i}; x_0^i\right), \overline{t}_{l,i}\right), \tag{15}$$

$$s = 1,2,\dots,k;$$

$$\sigma_i^l = \frac{\psi^*\left(\overline{t}_{l,i}+0; x_0^i\right)\cdot\left[f^l\left(x\left(\overline{t}_{l,i}\right), p^l\left(\overline{t}_{l,i}\right)\right) - f^{l+1}\left(x\left(\overline{t}_{l,i}\right), p^{l+1}\left(\overline{t}_{l,i}\right)\right)\right]}{{g^l}_x^*\left(x\left(\overline{t}_{l,i}\right), \overline{t}_{l,i}; \alpha\right)\cdot f^l\left(x\left(\overline{t}_{l,i}\right), p^l\left(\overline{t}_{l,i}\right)\right) + g_t^l\left(x\left(\overline{t}_{l,i}\right), \overline{t}_{l,i}; \alpha\right)}$$

$$l = 1,2,\dots,L-1, \tag{16}$$

is satisfied, where $\overline{t}_{l,i}$ is the moment of intersection of the trajectory of the system with the switching surface $g^l(x,t;\alpha) = 0$ under the initial condition x_0^i; $\psi\left(t; x_0^i\right)$ is the solution to the following adjoint system:

$$\dot{\psi}^*\left(t; x_0^i\right) = -\psi^*\left(t; x_0^i\right)\cdot\frac{\partial f^l\left(x(t), p^l(t)\right)}{\partial x}, t \in \Pi_l\left(x_0^i; \alpha, p\right) \tag{17}$$

$$\psi(T;\mathrm{x}_0^i) = -\frac{\partial}{\partial \mathrm{x}}\mathrm{J}_i\left(\mathrm{x}^i(T;\mathrm{x}_0^i,p,\mathrm{g}),p,\mathrm{g}\right),$$

satisfying the following jump condition at the time of intersection of the trajectory of the system (9) with the switching surface:

$$\psi(\bar{t}_{l,i}-0;\mathrm{x}_0^i) = \psi(\bar{t}_{l,i}+0;\mathrm{x}_0^i) - \sigma_i^l \cdot \mathrm{g}_\mathrm{x}^l\left(\mathrm{x}(\bar{t}_{l,i}),\bar{t}_{l,i};\alpha\right).$$

Here $\Pi_l(\mathrm{x}_0^i;\alpha,p)$, $l = 1,2,\dots,L$, $i = 1,2,\dots,N$, is the time interval during which the trajectory with the initial condition x_0^i and at the current values of the parameters (α,p) was in the zone X^l.

Summarizing the results of previous works, we obtained formulas for the gradient of the objective functional with respect to the vector of object parameters p:

$$\frac{\partial \mathrm{J}(p,\mathrm{g})}{\partial p^k} = \frac{1}{N}\sum_{i=1}^{N}\int_{\underline{t}_k}^{\bar{t}_k}\left[-\frac{\partial f\left(\mathrm{x}(t;\mathrm{x}_0^i)\right)}{\partial p^k}\cdot\psi(t;\mathrm{x}_0^i)\right]\mathrm{d}t, \qquad (18)$$

where $(\underline{t}_k,\bar{t}_k]$, $k = 1,2,\dots,L$, is the time during which the trajectory was in the region X^k, and therefore the vector p takes on value p^k.

The results of numerical experiments are presented by the example of solving several test problems.

The **third chapter** of the dissertation, which consists of 5 paragraphs, explores the problems of synthesis of control of nonlinear OLPs with various types of feedback on the input and output of an object and on various classes of zonal control functions given inaccurate information on the values of the object's parameters.

Consider the synthesis problem of controlling the dynamics of an object, a continuous and piecewise continuously differentiable function of the phase state $\mathrm{x}(t) \in \mathbb{R}^n$ of which is determined by a system of nonlinear differential equations:

$$\dot{\mathrm{x}}(t) = f(\mathrm{x}(t),\mathrm{u}(t),p),\ \ t \in (0,T]. \qquad (19)$$

Here $\mathrm{u}(t) \in U$ is the r-dimensional vector of piecewise continuous control actions, the range of admissible values of which is a closed convex set $U \subset \mathbb{R}^r$; p is the m-dimensional vector of time-constant parameters of the object whose exact values are not known, but can take values from some predetermined set P, moreover, the density function (weight) of the obtained values is given, defined on P:

$$0 \leq \rho_P(p) \leq 1,\ p \in P, \int_P \rho_P(p)\mathrm{d}p = 1; \qquad (20)$$

T is the duration of the control process. Vector-function $f(\mathrm{x}, \mathrm{u}, p)$ is continuously differentiable with respect to the first two arguments and continuous with respect to the third argument.

We will assume that the initial state of the object $\mathrm{x}^0 = \mathrm{x}(0)$ is not exactly specified, but there is a known set X^0 of possible initial states with a density (weight) function $\rho_{X^0}(\mathrm{x}^0)$ defined on X^0:

$$0 \leq \rho_{X^0}(\mathrm{x}) \leq 1, \mathrm{x}^0 \in X^0, \int_{X^0} \rho_{X^0}(\mathrm{x})\mathrm{dx} = 1. \qquad (21)$$

For each given specific initial point $\mathrm{x}^0 \in X^0$ and parameter values $p \in P$, we evaluate the quality of object control over a time interval $[0, T]$ using the functional:

$$\mathrm{I}(\mathrm{u}; T, \mathrm{x}^0, p) = \int_0^T f^0\big(\mathrm{x}(t), \mathrm{u}(t)\big)\mathrm{d}t + \Phi(\mathrm{x}(T), T), \qquad (22)$$

where $\mathrm{x}(t) = \mathrm{x}(t; \mathrm{x}^0, p, \mathrm{u})$ is the solution to the system of differential equations (19) under initial conditions $\mathrm{x}(0) = \mathrm{x}^0 \in X^0$, control $\mathrm{u}(t) \in U$ and parameter values $p \in P$.

The process completion time T can be either a given value, as in the case of functional (22), and an optimized value, as, for example, in performance problems. Considering that the initial state and parameter values are not specified exactly, but determined with the accuracy of the density functions on the corresponding set, we will evaluate the quality of object control by the following average value of functional (22) over all possible initial states $\mathrm{x}^0 \in X^0$ and parameter values $p \in P$:

$$\mathrm{J}(\mathrm{u}, T) = \frac{1}{\mathrm{mes}X^0 \cdot \mathrm{mes}P} \int_{X^0} \int_P \mathrm{I}(\mathrm{u}; T, \mathrm{x}^0, p)\, \rho_{X^0}(\mathrm{x}) \rho_P(p) \mathrm{d}p\mathrm{dx}^0. \qquad (23)$$

The dynamics of the process (19) is controlled taking into account the presence of feedback on the current state of the output of the object $\mathrm{y}(t)$, which is determined by the nonlinear function of its state $\mathrm{x}(t)$:

$$y(t) = G(x(t)),\ \ y \in \mathbb{R}^{\nu}, \quad (24)$$

where the ν-dimensional vector-function of observation $G(x)$ is continuously differentiable with respect to each variable on the set $X \subset \mathbb{R}^n$ of all possible states of the object, whose dynamics is described by system (19) under various initial states $x^0 \in X^0$, the values of the parameters $p \in P$ and of the controls $u(t) \in U$, $t \in (0, T]$.

Feedback (taking information about the output status) can be carried out both continuously at $t \in (0, T]$ and at discrete time points $\tau_j \in [0, T], j = 0,1,2, \dots, N$.

Values of the control actions $u(t)$ in the control process will be assigned as follows. Let $Y \subset \mathbb{R}^{\nu}$ be the set of values of the observed output vector $y(\cdot)$ from (24) under various possible admissible state values $x(\cdot) \in X$. We partition the set Y into L disjoint open subsets (zones) $Y^i \subset \mathbb{R}^{\nu}$ such that

$$Y = \bigcup_{i=1}^{L} \overline{Y}^i,\ Y^i \cap Y^j = \emptyset,\ i \neq j,\ i, j = 1,2, \dots, L, \quad (25)$$

where $\overline{Y}^i$ is the closure of the set Y^i. The boundaries between any two adjacent (having a common boundary) subsets Y^i and Y^j are determined by known continuous and almost everywhere differentiable functions $h_{ij}(y) = -h_{ji}(y) = 0$, and we assume that $Y^i \subset \{y: h_{ij}(y) < 0\}$ or $Y^i \subset \{y: h_{ji}(y) \geq 0\}$.

Values of the control actions $u(t)$ at current points of time in the process of controlling the dynamics of the object (19) will be assigned depending on which of the subsets Y^i the observed current value of the output vector $y(t)$ belongs to. Such controls will be called zonal.

The problem of assigning to each zone $Y^i, i = 1,2, \dots, L$, optimal (in the sense of functional (20)) control actions will be called the problem of synthesis of zonal control by output.

We consider four variants of the problem of synthesis of zonal controls are considered.

Problem 1. Given discrete time observations $\tau_j \in [0, T]$, $j = 0,1,2, \dots, N$, $\tau_0 = 0$, $\tau_N = T$, at which it is possible to measure the value of the output state of the object $\mathrm{y}(\tau_j) = G\left(\mathrm{x}(\tau_j)\right) \in Y$. Constant at $t \in [\tau_j, \tau_{j+1})$ the control values $\mathrm{u}(t)$ are determined depending on the last measured value of the vector of observations of the current output of the object, namely, depending on which subset (zone) Y^i, $i = 1,2,\dots,L$, of space Y the last measured (observed) output state belonged to:

$$\mathrm{u}(t) = \mathrm{v}^i = \text{const}, \ \mathrm{y}(\tau_j) = G\left(\mathrm{x}(\tau_j)\right) \in Y^i, t \in [\tau_j, \tau_{j+1}), \\ \mathrm{v}^i \in U \subset \mathbb{R}^r, \ i = 1,2,\dots,L, \ j = 0,1,\dots,N-1. \tag{26}$$

Problem 1 consists in finding admissible zonal control values v^i, $i = 1,2,\dots,L$, according to (26), which optimize the value of functional (23). The dimension of the optimized finite-dimensional vector in this case is $L \times r$.

Problem 2. Control actions are determined by linear functions of the results of observation over the parameters of the state of the output of the object at given discrete points of time $\tau_j \in [0,T]$, $j = 0,1,2,\dots,N$:

$$\mathrm{u}(t) = K_1^i \cdot \mathrm{y}(\tau_j) + K_2^i, \ \mathrm{y}(\tau_j) = G\left(\mathrm{x}(\tau_j)\right) \in Y^i, \\ t \in [\tau_j, \tau_{j+1}), i = 1,2,\dots,L, \ j = 0,1,\dots,N-1. \tag{27}$$

Here K_1^i is the matrix of dimension $r \times \nu$ and K_2^i is the r-dimensional vector, which are constants for $t \in [\tau_j, \tau_{j+1})$. Problem 2 consists in determining the admissible zonal values K_1^i and K_2^i, $i = 1,2,\dots,L$. The dimension of the optimized vector is $L \times r \times (\nu + 1)$.

Problem 3. Continuous measurements of the vector of observations of the output of the object are carried out, and control actions take zonal values:

$$\mathrm{u}(t) = \mathrm{w}^i = \text{const}, \mathrm{y}(t) = G(\mathrm{x}(t)) \in Y^i, t \in [0,T], \\ \mathrm{w}^i \in U \subset \mathbb{R}^r, \ i = 1,2,\dots,L. \tag{28}$$

In Problem 3, it is required to determine admissible zonal values of the control w^i, $i = 1,2,\dots,L$, optimizing the value of functional (23). The dimension of the optimized vector is $L \times r$.

Problem 4. Continuous measurements of the vector of observations of the output state of the object are carried out. The control is determined by a linear function of the measured output values:

$$\mathrm{u}(t) = K_1^i \cdot \mathrm{y}(t) + K_2^i,\ \mathrm{y}(t) = G\big(\mathrm{x}(t)\big) \in Y^i, t \in [0, T] \\ i = 1,2, \dots, L,\ j = 0,1, \dots, N-1. \quad (29)$$

It is required to find admissible values of K_1^i and K_2^i, $i = 1,2, \dots, L$, that optimize the value of functional (23). The number of optimized parameters is $r \times L \times (\nu + 1)$

We obtained formulas for the gradients of the functionals of the problems under consideration. The following theorems hold.

Theorem 3.1. The components of the gradient of the objective functional on the class of controls (26) are determined by the formulas:

$$\frac{\partial J}{\partial v^i} = \frac{\int_{X^0} \int_P \frac{\partial I(u; T, \mathrm{x}^0, p)}{\partial v^i} \rho_P(p) \rho_{X^0}(\mathrm{x}^0)\, \mathrm{d}P\, \mathrm{d}X^0}{mes(X^0) \cdot mes(P)},$$

$$\frac{\partial I(u; T, \mathrm{x}^0, p)}{\partial v^i} = \int_{\Pi_i(\mathrm{x}^0, p, u)} \left[\frac{\partial f^0(\cdot)}{\partial u} - \psi^T(t; \mathrm{x}^0, p, u) \frac{\partial f(\cdot)}{\partial u}\right] \mathrm{d}t,$$

where

$$\Pi_i(\mathrm{x}^0, p, u) = \bigcup_{j:\, G\left(\mathrm{x}(\tau_j; \mathrm{x}^0, p, u)\right) \in Y^i} [\tau_j, \tau_{j+1}), \qquad i = 1,2, \dots, L;$$

the function $\psi(t; \mathrm{x}^0, p, u)$ is the solution to the following adjoint Cauchy problem:

$$\psi(T; \mathrm{x}^0, p, u) = -\frac{\partial \Phi\big(\mathrm{x}(T; \mathrm{x}^0, p, u)\big)}{\partial \mathrm{x}},$$

$$\dot{\psi}^T(t; \mathrm{x}^0, p, u) = \frac{\partial f^0(\mathrm{x}(t; \cdot), p, u)}{\partial \mathrm{x}} - \psi^T(t; \mathrm{x}^0, p, u) \frac{\partial f(\mathrm{x}(t; \cdot), p, u)}{\partial \mathrm{x}},$$

for $t \in [0, T)$, under the observance of the condition (26).

Theorem 3.2. The components of the gradient of the objective functional on the class of controls (29) are determined by the formulas:

$$\frac{\partial \mathrm{J}(\mathrm{u})}{\partial K_1^i} = \int_{X^0} \int_P \int_{\Pi_i(\mathrm{x}^0, p, \mathrm{u})} \left[\frac{\partial f^0(\mathrm{x}(t; \mathrm{x}^0, p, \mathrm{u}), p, \mathrm{u})}{\partial \mathrm{u}} - \right.$$

$$-\psi^*(t;\mathrm{x}^0,p,\mathrm{u})\cdot\frac{\partial f(\mathrm{x}(t;\mathrm{x}^0,p,\mathrm{u}),p,\mathrm{u})}{\partial\mathrm{u}}\Bigg]^*\,\mathrm{d}t\times$$

$$\times G\big(\mathrm{x}(t;\mathrm{x}^0,p,\mathrm{u})\big)\frac{\rho_{X^0}(\mathrm{x})\,\rho_P(p)}{\mathrm{mes}\,X^0\times\mathrm{mes}\,P}\,\mathrm{d}p\,\mathrm{d}x^0,$$

$$\frac{\partial\mathrm{J}(\mathrm{u})}{\partial K_2^i}=\int_{X^0}\int_P\int_{\Pi_i(\mathrm{x}^0,p,\mathrm{u})}\Bigg[\frac{\partial f^0(\mathrm{x}(t;\mathrm{x}^0,p,\mathrm{u}),p,\mathrm{u})}{\partial\mathrm{u}}-$$

$$-\psi^*(t;\mathrm{x}^0,p,\mathrm{u})\cdot\frac{\partial f(\mathrm{x}(t;\mathrm{x}^0,p,\mathrm{u}),p,\mathrm{u})}{\partial\mathrm{u}}\Bigg]^*\,\mathrm{d}t\times$$

$$\times\frac{\rho_{X^0}(\mathrm{x})\rho_P(p)}{\mathrm{mes}\,X^0\times\mathrm{mes}\,P}\,\mathrm{d}p\,\mathrm{d}x^0$$

where $\Pi_i(\mathrm{x}^0,p,\mathrm{u})=\{t\in[0,T]:G\big(\mathrm{x}(t;\mathrm{x}^0,p,\mathrm{u})\big)\in Y^i\}$, $i\in\{1,2,\dots,L\}$; $\psi(t;\mathrm{x}^0,p,\mathrm{u})$ for $t\in[0,T]$ is the solution to the adjoint Cauchy problem:

$$\psi(T;\mathrm{x}^0,p,\mathrm{u})=-\frac{\partial\Phi(\mathrm{x}(T;\mathrm{x}^0,p,\mathrm{u}),T)}{\partial\mathrm{x}},$$

$$\dot\psi^*(t;\mathrm{x}^0,p,\mathrm{u})=\frac{\partial f^0(\mathrm{x}(t;\mathrm{x}^0,p,\mathrm{u}),p,\mathrm{u})}{\partial\mathrm{x}}-$$

$$-\psi^*(t;\mathrm{x}^0,p,\mathrm{u})\cdot\frac{\partial f(\mathrm{x}(t;\mathrm{x}^0,p,\mathrm{u}),p,\mathrm{u})}{\partial\mathrm{x}}+$$

$$+\left[\frac{\partial f^0(\mathrm{x}(t;.),p,\mathrm{u})}{\partial\mathrm{u}}-\psi^*(t;\mathrm{x}^0,p,\mathrm{u})\frac{\partial f(\mathrm{x}(t;.),p,\mathrm{u})}{\partial\mathrm{u}}\right]\times$$

$$\times K_1^i\times\frac{\partial G\big(\mathrm{x}(t;.)\big)}{\partial\mathrm{x}},\ \text{if}\ G\big(\mathrm{x}(t;\mathrm{x}^0,p,\mathrm{u})\big)\in Y^i,$$

satisfying the jump condition at the boundary of the zones:

$$\psi^*\big(\bar t_{ji}-0;\mathrm{x}^0,p,\mathrm{u}\big)=\psi^*\big(\bar t_{ji}+0;\mathrm{x}^0,p,\mathrm{u}\big)-$$

$$-\frac{\partial h_{ji}\left(\mathrm{y}(\bar t_{ji})\right)}{\partial G}\cdot\frac{\partial G\left(\mathrm{x}(\bar t_{ji};\mathrm{x}^0,p,\mathrm{u})\right)}{\partial\mathrm{x}}\cdot\sigma_{ji},$$

$$\sigma_{ji} = \frac{\psi^*(\bar{t}_{ji}+0;.)}{\frac{\partial h_{ji}\left(\mathrm{y}(\bar{t}_{ji})\right)}{\partial G}} \cdot \frac{\left[f\left(\mathrm{x}(\bar{t}_{ji};.),K^j,p\right) - f\left(\mathrm{x}(\bar{t}_{ji};.),K^i,p\right)\right]}{\frac{\partial G\left(\mathrm{x}(\bar{t}_{ji};.)\right)}{\partial \mathrm{x}} \cdot f\left(\mathrm{x}(\bar{t}_{ji};.),K^j,p\right)}.$$

Here $\bar{t}_{ji}$ are the points of time when the value of the observation vector (24) upon transition from one zone Y^j to another Y^i falls on their boundary, i.e. $h_{ji}\left(\mathrm{y}(\bar{t}_{ji})\right) = 0,\ \ i,j \in \{1,2,\dots,L\}$.

At the end of the third chapter, the results of numerical experiments are presented.

The **fourth chapter** of the dissertation consists of 4 paragraphs. For decision-making systems on controlling dynamic processes, which includes the stages of parametric identification of a mathematical model and optimization of regimes, we propose an approach that combines these stages. The result of this is obtaining a “locally optimal” model in the vicinity of the optimal regime.

Let the process be described by the following initial-value problem:

$$\dot{\tilde{\mathrm{x}}}(t) = \tilde{f}(\tilde{\mathrm{x}}(t),\mathrm{u},\mathrm{v},\mathrm{p}),\ \ t \in [0,T], \tag{30}$$

$$\tilde{\mathrm{x}}(0) = \tilde{\mathrm{x}}_0 \in X_0. \tag{31}$$

Here X_0 is the set of possible initial states of the process; $\tilde{\mathrm{x}}(t) = \tilde{\mathrm{x}}(t;\tilde{\mathrm{x}}_0,\mathrm{u},\mathrm{v})$ is the function that determines the process under given initial conditions $\tilde{\mathrm{x}}_0$, the values of the unregulated parameters $\mathrm{v} \in V \subseteq \mathbb{R}^m$ defined at the beginning of its course, and assigned (selected) values of regulated parameters $\mathrm{u} \in U \subseteq \mathbb{R}^r$; $\mathrm{p} \in P \subset \mathbb{R}^l$ is the vector parameters of the mathematical model; U is the set of admissible control parameters; V is the set of possible values of the unregulated parameters; P is the set of admissible values of the parameters of the mathematical model of the process.

Let the functional

$$\mathrm{J}(\mathrm{u};\tilde{\mathrm{x}}_0,\mathrm{v},\mathrm{p}) = \int_0^T \tilde{f}^0(\tilde{\mathrm{x}}(t),\mathrm{u},\mathrm{v})\mathrm{d}t + \Phi(\tilde{\mathrm{x}}(T),\mathrm{u},\mathrm{v}) \to \min_{\mathrm{u}\in U} \tag{32}$$

define the quality of the selected values of the vector of control parameters u depending on the given initial state $\tilde{x}_0$ and values of the vector of unregulated parameters v; $\tilde{f}^0(\cdot)$ and $\Phi(\cdot)$ are given functions that are continuously differentiable with respect to their first two arguments.

Suppose that the first stage of process modeling, the structural identification problem, has been solved due to, for example, some a priori qualitative information about the nature of the process, which is described by the following system:

$$\dot{x}(t) = f(x(t), u, v, p), \quad t \in [0, T], \tag{33}$$

where $f(\cdot)$ is the n-dimensional vector-function continuously differentiable with respect to its arguments given to within the parameters p, which in most cases differs from the function that actually describes the process $\tilde{f}(\cdot)$; p is the vector of parameters of the mathematical model of the process, the values of which must be determined at the stage of parametric identification.

To carry out parametric identification, it is necessary to have observations of the state of the technological process, which may have a different character. To carry out parametric identification, it is necessary to have observations over the state of the technological process, which may have a different character. For example, for given values of unregulated and regulated parameters $v^i \in V$ and $u^i \in U$, there may be observations of the state of the process at given points of time $t_{ij} \in [0, T]$:

$$\begin{gathered} \hat{x}^i_j = \hat{x}^i(t_{ij}; u^i, v^i), \quad t_{ij} \in [0, T], \\ j = 0,1, \dots, M_i; \quad i = 1,2, \dots, N. \end{gathered} \tag{34}$$

or, in particular, only at the initial and final points of time $t_{i0} = 0$, $t_{iM_i} = T$:

$$\hat{x}^i_0 = \hat{x}^i(0; u^i, v^i), \quad \hat{x}^i_T = \hat{x}^i(T; u^i, v^i), i = 1,2, \dots, N, \tag{35}$$

where N is the number of observations of separately occurring processes; M_i is the number of observations of the state at each process, for each of which a positive weighting coefficient γ_i, $i = 1,2, \dots, N$, is set, whose values are determined by the degree of reliability and accuracy of the results of the observations, and, as a rule, $\gamma_i \in [0,1]$, $i = 1,2,\dots,N$.

The parametric identification problem of determining the parameters p of the model using, for example, the least squares criterion for observations of the form (34) leads to minimization of the functional

$$S_1(\mathrm{p}) = \sum_{i=1}^{N} \sum_{j=1}^{M_i} \gamma_i \left\| \mathrm{x}^i\left(t_{ij}; \mathrm{u}^i, \mathrm{v}^i, \mathrm{p}\right) - \hat{\mathrm{x}}_j^i \right\|_{\mathbb{R}^n}^2 + \varepsilon \left\| \mathrm{p} - \tilde{\mathrm{p}}_0 \right\|_{\mathbb{R}^l}^2 \quad (36)$$

where ε and $\tilde{\mathrm{p}}_0$ are regularization parameters of the minimized functionality.

The statements of parametric identification problem (33) – (36) and of optimal control problem (32) – (33) belong to the same class of parametric optimal control problems, for the solution of which first-order optimization methods can be applied.

In the work, we propose, after the values of the initial conditions $\overline{\mathrm{x}}_0$ and unregulated parameters $\overline{\mathrm{v}}$ have already been set, to combine the stages of identification of the parameters p and optimization of control parameters u.

It is proposed to carry out parametric identification of the model after each iteration on optimizing the control parameters u, while all available observations used for the parametric identification problem are assigned a weight $\rho^i\left(\hat{\mathrm{x}}_0^i, \mathrm{v}^i, \mathrm{u}^i; \overline{\mathrm{x}}_0, \overline{\mathrm{v}}, \mathrm{u}^k\right)$, $i = 1,2,\dots,N$, the values of which are inversely proportional to the distance values of the observed process parameters $\hat{\mathrm{x}}_0^i, \mathrm{v}^i$, and u^i, from the specified values of the initial condition $\overline{\mathrm{x}}_0$, unregulated parameter $\overline{\mathrm{v}}$, and current value of the iterative process for optimization of the parameter u^k.

Another implementation of the proposed approach to combining the solution to the parametric identification problem and optimal control is to cut off from the set of observed values of the process parameters those observations whose values are away from the current vector $(\overline{\mathrm{x}}_0, \overline{\mathrm{v}}, \mathrm{u}^k)$ by a distance greater than some given value, at which weighting functions are not used.

When using the proposed approach in automated process control systems, there is no need to memorize a mathematical model; it is replaced by the need to preserve the so-called information model of the process, which consists of the form of differential equations (33), of the form of the optimized functional (32), and of the values of the observations of the process state parameters (34) or (35).

The results of numerical experiments using the software developed by the author are presented.

In the **fifth chapter** of the dissertation, which consists of 5 paragraphs, we study coefficient-inverse problems and the problems of synthesis of control of ODPs.

The first three paragraphs of the fifth chapter provide a statement and an approach to solving the problem of identifying the nonlinear hydraulic resistance coefficient of a main pipeline section during transportation of hydrocarbon raw materials under an unsteady fluid motion mode. The proposed approach is that the entire section is divided into a given number of subsections, the ends of which are optimizable, and the concept of a "zone" is introduced for the optimizable parameters of the identifiable function, which depends on the current state of the object. The set of all phase states of the object is divided into a finite number of subsets (zones), in each of which the identifiable function is assumed to be constant, linear, or having some other form of a functional dependence. As a result, the identification problem under consideration is reduced to a class of finite-dimensional optimization problems, for the solution of which it is proposed to use efficient numerical first-order finite-dimensional optimization methods. We derive formulas for the gradient components of the objective functional in the space of identifiable parameters. The obtained values of the optimizable vector can then be used to construct the identifiable function from any class of functions using interpolation and approximation techniques.

The unsteady motion of an incompressible fluid of constant density ρ and viscosity υ along a linear horizontal section of an oil pipeline of length ℓ and diameter d is adequately described by the following system of differential equations of hyperbolic type:

$$-\frac{\partial p}{\partial x}=\rho\left[\frac{\partial \omega}{\partial t}+\alpha\lambda\omega\right],\quad -\frac{\partial p}{\partial t}=c^2\rho\frac{\partial \omega}{\partial x}, \tag{37}$$

$$x\in X=(0,\ell),\ \ t>t_0,$$

where $p=p(x,t)$ and $\omega=\omega(x,t)$ are the pressure and fluid velocity at the point $x\in(0,\ell)$ of the pipeline at the point of time $t>t_0$; c is the sound velocity in a liquid; α is the linearization coefficient. It is known that the hydraulic resistance coefficient λ depends on the regime of fluid motion, i.e. from the Reynolds number $\mathrm{Re}=\omega d/\upsilon$ and the relative roughness $\varepsilon=k/d$ of the inner surface of the pipeline section, where k is the absolute roughness. Assuming the roughness is different for different points of the pipeline section, we conclude that the value λ is a function of speed ω and the point of the pipe x, i.e. $\lambda=\lambda(\omega,x)$.

The initial data for solving the parametric inverse problem of identifying the hydraulic resistance coefficient are the pressure and/or fluid velocity regimes observed at various points $\overline{x}_i$, $i=1,2,\dots,M$ of a linear section of the pipeline continuously or at discrete points of time. The problem under consideration is formulated in a variational formulation within the framework of the optimal control problem of the ODP, and more precisely the control synthesis problem, because the current value of the identifiable function depends on the current state of the process. In the problem, we optimize the functional

$$\mathrm{J}(\lambda)=\frac{1}{N}\sum_{i=1}^{N}\int_{\underline{T}_i+\Delta T}^{\overline{T}_i}\{[\omega(0,t;\lambda,\varphi_{0i},\varphi_{li})-\psi_{0i}(t)]^2+ \tag{38}$$

$$+[\omega(\ell,t;\lambda,\varphi_{0i},\varphi_{li})-\psi_{li}(t)]^2\}\mathrm{d}t$$

of the mean-square deviation of the observed boundary conditions

$$\omega_i(0,t)=\psi_{0i}(t),\ \ \omega_i(\ell,t)=\psi_{li}(t), \tag{39}$$

from those calculated as a result of solution to the problem (37) with boundary conditions

$$p_i(0,t)=\varphi_{0i}(t),\ \ p_i(\ell,t)=\varphi_{li}(t),\ i=1,2,\dots,N, \tag{40}$$

and with initial conditions

$$p_i(x, \underline{T_i}) = [\varphi_{li}(\underline{T_i}) - \varphi_{0i}(\underline{T_i})]\, x/\ell + \varphi_{0i}(\underline{T_i}),$$
$$\omega_i(x, \underline{T_i}) = [\varphi_{0i}(\underline{T_i}) - \varphi_{li}(\underline{T_i})]\, /(\beta\, \ell). \tag{41}$$

Here $[\underline{T_i}, \overline{T}_i]$, $i = 1,2, \ldots, N$, are time intervals of a sufficiently long duration in which observations were made of the pumping regimes; ΔT is the specified duration of the influence of the initial conditions for each specific site, taking into account the properties of the pumped liquid and the geometric dimensions of the site.

Assume that, based on a priori information about the process, the range of realistically possible values of the fluid velocity in the oil pipeline is known, i.e. $\omega(x,t) \in \Omega = [\underline{\omega}, \overline{\omega}]$, $x \in (0, \ell)$, $t > t_0$, где $\underline{\omega}$ and $\overline{\omega}$ are known limiting values of the function $\omega(x,t)$. We quantize the sets Ω and X by a group of predefined values ω_i, $i = 0,1,2, \ldots, L_\omega$, and x_j, $j = 0,1,2, \ldots, L_x$, such that:

$$\underline{\omega} = \omega_0 < \omega_1 < \cdots < \omega_{L_\omega} = \overline{\omega},$$
$$0 = x_0 < x_1 < \cdots < x_{L_x} = \ell.$$

We assume that the function $\lambda(\omega, x)$ is piecewise constant with respect to ω and x:

$$\lambda(\omega, x) = \lambda_{ij} = \text{const},\ \omega \in \Omega_i = [\omega_{i-1}, \omega_i),$$
$$x \in X_j = [x_{j-1}, x_j);\ i = 1,2, \ldots, L_\omega, j = 1,2, \ldots, L_x. \tag{42}$$

We also considered a more general case when the function $\lambda(\omega, x)$ is determined by means of a predetermined set of basis functions, i.e.

$$\lambda(\omega, x) = \sum_{s=1}^{M} \lambda_{ij}^{s}\, \gamma_s(\omega)\, , \omega \in \Omega_i,\ x \in X_j.$$

The problem of finding the function $\lambda(\omega, x)$ is replaced by the problem of finding a finite-dimensional vector $\Lambda = (\lambda_{ij})_{i=1,2,\ldots,L_\omega}^{j=1,2,\ldots,L_x}$ that minimizes the functional (38). To this end, we obtained formulas for the gradient components of the objective functional with respect to the identifiable parameters. For numerical solution to the inverse coefficient-inverse problem, software was developed and numerical experiments were conducted by the example of solving several test problems.

The results obtained can also be used in mathematical modeling and solving inverse problems for many technological processes and technical objects, the identifiable parameters of which are functions of the state of the process (object).

In the fourth and fifth paragraphs of the fifth chapter, we study two synthesis problems of feedback control for ODP on the basis of continuous monitoring of the phase state of an object at its defined points.

We solve the problem of synthesis of the control of the heating process of a tubular heat exchanger in a steam jacket with feedback, described by the equation:

$$u_t(x,t) + a u_x(x,t) = -\alpha[u(x,t) - \vartheta(t)], \qquad (x,t) \in \Omega = [0,\ell] \times (0,T], \tag{43}$$

where $a = \text{const}$ is the given fluid velocity in the heat exchanger and heating system; ℓ is the length of the part of the heat exchanger in the steam jacket; $\vartheta(t)$ is the temperature created inside the steam jacket, which is a piecewise continuous function; $\alpha = \text{const}$ is the heat transfer coefficient; $u(x,t)$ is the fluid temperature at the point $x \in (0,\ell)$ at the point of time t from the class of continuously differentiable with respect to x and t functions.

The initial and boundary conditions are given in the following form:

$$u(x,t_0) = u_0(x) \in G_0, \;\; x \in [0,\ell], \tag{44}$$

$$u(0,t) = (1-\gamma)\, u(\ell, t-\Delta), \;\; t \in (0,T], \tag{45}$$

where $\gamma \in G_1 = (0,\delta), 0 < \delta < 1$, is the parameter characterizing the amount of heat loss during the passage of fluid through the heating system; Δ is the transport lag determined by the length L of the heating system outside the steam jacket ($\Delta = L/a$). Here, the continuous function $u_0(x)$ and the parameter γ are given inaccurately, but their values belong to some given sets G_0 and G_1, with known distribution density functions $\rho_{G_0}\big(u_0(x)\big)$ and $\rho_{G_1}(\gamma)$.

Inside the furnace, thermocouples (sensors) are installed along the heat exchanger, which measure the temperature of the fluid at given N points $\overline{x}_j \in [0, \ell]$, $j = 1,2, \dots, N$. These measurements are used to determine the required temperature in the steam jacket $\vartheta(t)$. The sensors implement operational monitoring and input of information about the state of the heating process at these points into the control system defined by the vector

$$\overline{u}(t) = \big(u(\overline{x}_1, t), u(\overline{x}_2, t), \dots, u(\overline{x}_N, t)\big),\ t \in (0, T]. \qquad (46)$$

To control the process of heating the fluid in the steam jacket, we synthesize a regulator, which, according to the results of temperature measurements in points $\overline{x}_j \in [0, \ell]$, $j = 1,2, \dots, N$, of the heat exchanger, maintains the output temperature $u(\ell, t)$ of the fluid at the given level by setting the required temperature $\vartheta(t)$ in the steam jacket. We are given a set V of admissible controls $\vartheta(t)$, which is determined based on technological conditions and, as a rule, it is determined by an inequality of the form

$$\underline{\vartheta} \le \vartheta(t) \le \overline{\vartheta},\ \ t \in [0, T], \qquad (47)$$

where $\underline{\vartheta}$ and $\overline{\vartheta}$ are given constants.

The considered problem of controlling the process of heating the fluid with feedback consists in constructing the dependence of the temperature of the furnace on the measured state values at the observed points

$$\vartheta(t) = w\big(\overline{u}(t)\big) \in V, t \in (0, T], \qquad (48)$$

minimizing the quality control criterion, given, for example, in the form of the following functional:

$$\mathrm{J}(w) = \int_{G_0} \int_{G_1} \int_0^T [u(\ell, t; w, u_0,\ \gamma) - \tilde{u}(t)]^2 \mathrm{d}t\ \mathrm{d}\rho_{G_1}(\gamma) \mathrm{d}\rho_{G_0}(u_0). \qquad (49)$$

Here $u(x, t; w, u_0,\ \gamma)$ is the solution to the problem (43) – (45), corresponding to the selected initial and boundary conditions $u_0(x)$ and γ, as well as to admissible control values $w\big(\overline{u}(t)\big)$; $\tilde{u}(t)$ is the a function that characterizes the desired temperature of the fluid at the right end (exit) of the heat exchanger during the entire process of heating the fluid.

Suppose that, based on the technological conditions, it is known that under all sorts of admissible control values, of initial and boundary conditions, the temperature of the fluid in the heat exchanger satisfies the following inequality:

$$\underline{\underline{u}} \leq u(x, t; w, u_0, \gamma) \leq \overline{\overline{u}}, \ \ (x, t) \in \Omega. \qquad (50)$$

We partition the set of all possible temperature values $\left[\underline{\underline{u}}, \overline{\overline{u}}\right]$ into M semi-intervals

$$\left[\underline{\underline{u}}, \overline{\overline{u}}\right] = \bigcup_{k=1}^{M} [u_{k-1}, u_k), \ \ u_0 = \underline{\underline{u}}, \ \ u_M = \overline{\overline{u}}, \qquad (51)$$

by given values u_k, $k = 0,1,2, \dots, M$. To determine the control of the form (48), we use the class of piecewise constant functions:

$$\begin{gathered} w\left(\overline{u}(t)\right) = \omega_{i_1, i_2, \dots, i_N} = \text{const}, \\ u_{i_j - 1} \leq u\left(\overline{x}_j, t; \vartheta, u_0, \gamma\right) \leq u_{i_j}, \qquad t \in (0, T], \\ i_j \in \{1,2, \dots, M\}, \ \ j = 1,2, \dots, N. \end{gathered} \qquad (52)$$

where $\omega_{i_1, i_2, \dots, i_N}$ are yet unknown values that need to be determined from the condition of minimum of the functional (49).

Thus, the considered problem of controlling the heating process in the heat exchanger (43) – (45) with feedback (46) on the class of zonal piecewise constant functions (52) consists in determining the M^N-dimensional vector $\omega = \left(\omega_{i_1, i_2, \dots, i_N}\right)$, $i_j = 1,2, \dots, M$, $j = 1,2, \dots, N$, that minimizes the functional (49). To solve it, we obtained formulas for the gradient components of the objective functional. The results of numerical experiments on test problems are presented.

An important role in determining the optimal zonal values of the control function $\vartheta(t)$ using the approach described above is played by the choice of both the number and specifically the zones themselves. The following approach is recommended, in which an initial value of M is first randomly chosen and some zones are assigned. Having solved the above control synthesis problem, the analysis of the obtained optimal zonal values of the control action for all neighboring zones is carried out. If the optimizable parameters in any two adjacent zones differ slightly, then these adjacent zones can be combined into one larger zone, thus reducing the number M, and hence the number of switchings when controlling the furnace. If the optimizable parameters in any two adjacent zones differ significantly, then, on the contrary, each of these adjacent zones should be divided, for example, into two smaller zones, i.e. increase the number M, and again solve the control synthesis problem. The increase in the number of zones should be carried out until the value of the objective functional ceases to change (decrease) significantly.

We also considered the problem of controlling the heating process of a plate by means of concentrated point sources:

$$u_t = a^2 \Delta u + \sum_{j=1}^{L} \vartheta^j(t)\, \delta\left(x - \overline{x}^j\right), \quad (x,t) \in \Omega \times (0,T]. \tag{53}$$

Here Ω is the two-dimensional region occupied by the plate; at the points $\overline{x}^j = \left(\overline{x}_1^j, \overline{x}_2^j\right)$ of this plate there are placed heat sources with optimized power $\vartheta^j(t)$, $j = 1,2,\dots,L$; L is the given number of sources; Δ is the two-dimensional Laplace operator; $\delta(\cdot)$ is the two-dimensional generalized Dirac function; a^2 is the thermal diffusivity coefficient.

Suppose that on the plate at N points with coordinates

$$\tilde{x}^s = (\tilde{x}_1^s, \tilde{x}_2^s) \in \Omega, \;\; s = 1,2,\dots,N, \tag{54}$$

there are installed sensors that carry out operational monitoring and input into the control system of information about the state of the heating process at these points, determined by the vector:

$$\tilde{u}(t) = \left(\tilde{u}^1(t),\, \tilde{u}^2(t), \dots, \tilde{u}^N(t)\right), \\ = \left(u(\tilde{x}^1,t),\, u(\tilde{x}^2,t), \dots, u(\tilde{x}^N,t)\right),\; t \in (0,T], \tag{55}$$

$$u(x,t_0) = g_0(x) \in G_0(x), \;\; x \in \Omega, \tag{56}$$

$$u(x,t)|_{\Gamma_1} = \mathrm{g}_1(t) \in G_1(t), \;\; t \in (0,T], \tag{57}$$

$$\frac{\mathrm{d}}{\mathrm{d}n} u(x,t)|_{\Gamma_1} = \mathrm{g}_2(t) \in G_2(t), \;\; t \in (0,T]. \tag{58}$$

Here $G_i(\cdot)$, $i = 0,1,2$, are point-multiple mappings for which a closed bounded set is associated with each argument value, and corresponding distribution functions $\Phi_i\big(\mathrm{g}_i(\cdot)\big)$, $i = 0,1,2$, are specified that characterize the distribution of possible values accepted by the initial-boundary conditions.

The considered problem of controlling the heating process of the plate is to select admissible values of the power of the sources depending on the values of current states at the observed points of the plate

$$\vartheta^j(t) = \vartheta^j\big(\tilde{u}^1(t),\, \tilde{u}^2(t), \dots, \tilde{u}^N(t)\big), \;\; \vartheta^j(t) \in V^j, \\ j = 1,2,\dots,L; \;\; t \in (0,T]; \tag{59}$$

$$\tilde{u}^s(t) = u(\tilde{x}^s, t), \;\; s = 1,2,\dots,N, \tag{60}$$

minimizing the given functional, where V^j is the set of possible values, $V = (V^1, V^2, \dots, V^L)$.

The objective functional is as follows:

$$\mathrm{J}(\vartheta) = \int_{G_0}\int_{G_1}\int_{G_2} \mathrm{I}(x,T;\vartheta,\mathrm{g}_0,\mathrm{g}_1,\mathrm{g}_2)\, d\Phi_2(g_2)\, d\Phi_1(g_1)\, d\Phi_0(g_0) \\ + \sum_{j=1}^{L} \int_{t_0}^{T} \big[\vartheta^j(t)\big]^2\, \mathrm{d}t, \tag{61}$$

$$\mathrm{I}(\cdot) = \int_{\Omega} [u(x,T;\vartheta,\mathrm{g}_0,\mathrm{g}_1,\mathrm{g}_2) - \hat{u}(x)]^2 \mathrm{d}\Omega,$$

where $u(x,T;\vartheta,\mathrm{g}_0,\mathrm{g}_1,\mathrm{g}_2)$ is the solution to the problem (53), (56) – (58), corresponding to the specifically selected initial-boundary functions $\mathrm{g}_0(\mathrm{x}), \mathrm{g}_1(t)$, and $\mathrm{g}_2(t)$, and to admissible control values

$\vartheta(t)$; $\hat{u}(x)$ is a predetermined function characterizing the desired temperature distribution at the final time of the heating process.

Let it be known that the values of the phase states of the plate satisfy the inequality

$$\underline{u} \le u(x,t;\vartheta,g_0,g_1,g_2) \le \overline{u},\ (x,t) \in \Omega, \qquad (62)$$

under all possible admissible controls and initial-boundary functions

$$\vartheta(t) \in V,\ g_0(x) \in G_0, g_1(t) \in G_1, g_2(t) \in G_2. \qquad (63)$$

We partition the set of all possible temperature values $[\underline{u},\overline{u}]$ by the values $u_k,\ k=0,1,\dots,m$, into m temperature intervals:

$$[\underline{u},\overline{u}] = \bigcup_{k=1}^{m} [u_{k-1},u_k)\,,\ u_0 = \underline{u},\ u_m = \overline{u}. \qquad (64)$$

Piecewise constant control values will be selected depending on whether the temperature belongs to a particular temperature range.

Let the controls satisfy the condition of piecewise constant functions

$$\vartheta^j(t) = \vartheta^j_{i_1,i_2,\dots,i_N} = \text{const},\ i_s = 1,2,\dots,m; \quad s = 1,2,\dots,N;\ \ j = 1,2,\dots,L. \qquad (65)$$

in cases where the values of current states at the observed points correspond to the inequalities:

$$u_{i_s-1} \le u(\tilde{x}^s,t;\vartheta(t),g_0,g_1,g_2) \le u_{i_s}, \quad i_s = 1,2,\dots,m;\ \ s = 1,2,\dots,N. \qquad (66)$$

In the N-dimensional phase state space $u(\tilde{x}^s,t)$, $s=1,2,\dots,N$, the sets (66) represent N-dimensional parallelepipeds, the total number of which is equal to m^N.

It is clear that controls (65), as well as (59), assume the presence of feedback; and in case (65), the power values of the controlled sources during plate heating change only at the moments when the set of states at the observed points passes from one phase parallelepiped (66) into another.

The considered problem of controlling plate heating using feedback on the class of piecewise constant functions is to optimize the $L \times m^N$-dimensional vector

$$\vartheta = \left(\vartheta^j_{i_1,i_2,\dots,i_N}\right),\ i_s = 1,2,\dots,m,\ s = 1,2,\dots,N, \quad j = 1,2,\dots,L. \qquad (67)$$

For the synthesis of zonal controls, we obtained formulas for the gradient of the objective functional in the space of optimized parameters.

The **sixth chapter** of the dissertation, consisting of 4 paragraphs, is dedicated to the creation of algorithms and software to solve complex optimization problems based on interactive and automatic systems of control of a package of unconditional optimization programs. We carried out an analysis of methods and algorithms for managing the computational process for solving complex problems using multiprocessor (multi-core) and multi-node computer systems. The dialog system and the system of automatic control of an optimization process developed by the author are created according to a modular principle, taking into account further expansion of capabilities, and use algorithms for parallelizing calculations. The developed systems are equipped with an extensive library of optimization programs, the dialog service provided in them allows the user to control the process of solving problems on a computer, depending on the current results of calculations, choose the most rational sequence of the methods used, adjust the parameters of the methods if necessary, and make changes to the formulation of the problems being solved. Note that starting from the 70s of the last century, comprehensive work has been done in different schools to create intelligent algorithms for managing software packages. Here it is worth noting, for example, the school of Yu.G. Evtushenko, who developed control dialogue systems for unconditional optimization ("DISO") and optimal control ("DISOPT"), as well as the school of K.R. Aida-zade, who developed control systems for solving vector optimization problems ("DIVO") and global optimization ("GLOPT").

It is known that when using numerical optimization methods for solving specific problems, a huge amount of calculations usually has to be carried out. Therefore, the widespread introduction of optimization methods became possible only relatively recently, thanks to the creation of modern powerful high-speed computer systems. In spite of a large number of methods for the numerical solution of various classes of problems, the choice of the most effective method for solving a specific problem under certain values of its parameters requires a large number of comparative experiments. End users, as a rule, experience difficulties in conducting such experiments, which requires knowledge of the field of applicability of various numerical methods and the proper conduct of a comparative analysis of the results, which usually takes a lot of time. The quality of a numerical method is characterized by many factors: the area of convergence of the method, the rate of convergence, the execution time of one iteration, the amount of machine memory required to implement the method, the class of the problem being solved, etc. Optimization problems also have a great variety: among them there are smooth and non-smooth problems, low and high dimensions, ravine type, unimodal and multimodal (multi-extremal), etc. It is quite clear that not the search for a universal method, but a reasonable combination of various methods will make it possible to solve a given optimization problem with the greatest efficiency.

An optimization process is usually managed statically, when the user pre-sets the calculation scenario, that is, determines the sequence of methods used by him, their parameters, enters this information into a computer, and receives the results only after all calculations are completed. This approach, as a rule, leads to a significant amount of calculations, since it is difficult to predict in advance the course of the computational process in the formulation of the problem. It is more expedient to manage calculations in an interactive mode, when the user receives information about the current results in the process of calculations, changes the parameters of the method, and makes a purposeful and conscious transition from one optimization method to another. Dialogue systems and automatic control systems make it possible to create a universal tool for solving various practical problems.

For the class of problems of multidimensional unconditional optimization, we propose two approaches to facilitate the use of available application software packages using modern multiprocessor (multi-core) computer systems. One of such approaches involves the active work of the end user with the optimization software package in the interactive (dialogue) mode. Another approach involves managing packages using a specially designed management program in automatic mode.

Implementation for a sequential (single-core) architecture has an important independent meaning and can be considered as the basic block of implementations on multiprocessor (multicore) and multi-node architectures. Below, we describe in detail one of the possible schemes for implementing the algorithm for solving optimization problems on such architectures. Let M_1, M_2, …, M_k be a list of optimization methods made up of algorithms within the optimization software package. It is advisable to include methods of different characteristics if, generally speaking, little is known about the structure of the objective function. The list can also be problem-oriented if there is any information about the objective function, for example, it is known that the function is convex, quadratic, ravine, non-differentiable, etc. Progress in solving the problem is carried out in stages, each of which consists of training and working steps. The first of these stages is designed to identify a locally efficient algorithm from the available list of algorithms. After that, a working step is carried out, which consists in solving the problem using only the algorithm that was identified at the first stage. Both the training and the working steps are given certain time slices.

At the training stage, we can use two options:

1. To determine the local efficiency of methods, optimization is carried out from the same point. In this case, there is a somewhat uneconomical waste of computer time, and training is used only to identify a locally efficient algorithm.
2. The training time is used not only to find an effective algorithm, but also to move to the minimum point, since for training each next algorithm, not the initial, but the current point is used.

The second option is more economical and faster due to the fact that the training time is active in the optimization process. At the training stage, all the algorithms from the initial list M_1, M_2, …, M_k are given the opportunity to prove themselves during a given initial time slice. The only exceptions are those methods that turned out to be the least effective twice in a row at the training stage. They are not given a time slice and are temporarily excluded from the list. The value of the initial time slice depends on the type of function to be minimized, more precisely, on the time spent by the computer system for one computation of the objective function, and on the number of its variables: $\tau = \tau(n, \theta)$, where θ is the time per one computation of the function and n is the dimension of the optimization problem. As a result, at the training stage, the most effective algorithm is identified, which is used during the working step. The duration of the working step $T_\mathrm{i} = T_\mathrm{i}(n, \tau, \theta)$ is selected as follows:

$$T_0 = \alpha\tau,\ T_i = \delta_i T_{i-1} + T_0,\ \alpha > 1. \quad (68)$$

The value α depends on the complexity and dimension of the objective function and is chosen a priori. It is desirable to carry out the tests at different values of α. The value $\delta_\mathrm{i} = 1$ if the same method turned out to be the most effective at two successive stages. Otherwise, $\delta_\mathrm{i} = 0$, i.e. the duration of the working step can increase if any method has proved to be the most effective in several successive stages. This most likely means that to minimize the objective function, we found the method that reaches the optimum point in the shortest time, and therefore, for such a case (generally speaking, ideal), it makes no sense to perform further training. To calculate the values of local efficiencies of the methods, the following formula is used:

$$E_i = \frac{|f(\mathrm{x}^{k+1}) - f(\mathrm{x}^k)|}{|f(\mathrm{x}^k)| + \epsilon} + \frac{\|\mathrm{x}^{k+1} - \mathrm{x}^k\|}{\|\mathrm{x}^k\| + \epsilon} \quad (69)$$

Here E_i is the local efficiency of the i^{th} algorithm; x^k and $\mathrm{x}^{\mathrm{k}+1}$ are the initial and final points obtained using the i^{th} algorithm; $f(\mathrm{x}^\mathrm{k})$ and $f(\mathrm{x}^{\mathrm{k}+1})$ are the values of the objective function at these points; $\|\cdot\|$ is the Euclidean norm; and ϵ is a small positive number. If the value of the objective function has not decreased in a time slice, the local efficiency of such an algorithm is considered equal to 0. If, at the training stage, all methods from the list have found zero efficiency, further search is terminated, the whole process is stopped. This situation is possible in the case when the list of methods is not focused on solving the given problem (for example, the function is ravine, and the list consists of coordinate descent and gradient methods). Thus, the inclusion of diverse methods in the list will help to avoid such situations. Note that the local zero efficiency of a method can appear in a method even in the case when an arithmetic interrupt appears during its operation. Having introduced the handling of exceptions into the program code of the system, in this case it is necessary to ensure the transition from the $\mathrm{i^{th}}$ to the next, $(\mathrm{i}+1)^{\mathrm{th}}$ method from the list. At that, the $\mathrm{i^{th}}$ method does not complete the time slice allocated to it and receives the value of local efficiency $E_{\mathrm{i}} = 0$. The criterion for terminating the proposed procedure is the fulfillment of the search end condition in one of the methods. In this case, it is not a forced interruption from the timer that occurs, but a natural termination of the method and the entire procedure as a whole. In conclusion, the user receives the accumulated information about the search progress, which includes the optimal chain of methods that worked at the working steps, the total time to find a solution, as well as the values of the objective function, the design vector, and local efficiencies obtained at the training stages.

Consider a multithreaded approach: there are several independent instruction streams executed in parallel, accessing shared memory. The simplest implementation option seems to be an approach in which threads independently perform operations of the sequential algorithm described above. The solution to the optimization problem is carried out in stages. At each stage, the following actions (steps) are taken.

1. At the initial step, from the list of all available optimization algorithms – $M_1, M_2, \dots, M_{\mathrm{k}}$, several algorithms are randomly selected, e.g. $M_{s_1}, M_{s_2}, \dots, M_{s_N}$, the number N of which is chosen equal to the number of cores installed in the system. Note that the value of N can also be taken as a multiple of the number of CPU cores installed in the system (for example, $2N$, $3N$, etc.).
2. To determine the local efficiency of the methods, optimization is carried out from the same starting point x^0 and is used to identify the local efficiency of each algorithm that has worked. This step is used not only to find effective algorithms, but also to

simultaneously move to the minimum point, i.e. “working step time” is active in the optimization process. At the stage of the "working step", all algorithms from the initial list – $M_{s_1}, M_{s_2}, \dots, M_{s_N}$ – are given the opportunity to work out during a given initial time slice τ.

3. At the stage of the working step, the most efficient algorithms are identified. The duration of the working step T_i is chosen according to the formula (1) and can be increased if any method has proved to be the most effective at several successive stages. This means that to minimize a given objective function, we have found the method that reaches the optimum point in the shortest time, and therefore, for such a case, generally speaking, ideal, it makes no sense to carry out further sampling of algorithms.
4. The values of the local efficiencies E_i of the methods are calculated according to the formula (2). Half of those with the lowest efficiency are excluded from the list of working algorithms. At the same time, these algorithms build up a “low efficiency flag”. If, at the training stage, all methods from the list have found zero efficiency, further search stops, and the procedure for solving the optimization problem terminates.
5. The same number of other algorithms are added to the list of working algorithms as were excluded in the previous step. The list includes only those algorithms for which the low efficiency flag is the smallest in value (again, they are selected randomly). After forming a new list of working algorithms, steps 2–5 are repeated.

In the procedure described above, all the methods from the list are forced to interrupt after a specified time slice, while for some method(s) the next iteration that has already begun may not be completed. A possible modification of this procedure is to enable all the methods to complete the already started iterations or to carry out a whole number of iterations in the vicinity of the specified time interval. The criterion for exiting the described procedure is the zero efficiency of all the methods from the list, i.e. when a further search for the optimal point by any algorithm from the list does not lead to an improvement in the results.

Distributed systems have a hierarchical organization: they consist of heterogeneous nodes, each of which can, in turn, be a multi-processor/multi-core system, that is, one of the systems discussed above. It is natural to assume that maximum efficiency is achieved when the computational process is organized in accordance with this hierarchy. With this approach, at each of the nodes, calculations are performed according to the scheme most suitable for a given node. In fact, on each of the nodes of the distributed system, a separate application is executed – a "solver", which performs the operations of the selected optimization algorithm. Interaction between several applications is organized at the next level of the hierarchy through a dedicated central control process – a "supervisor". The first stage in solving a problem in a distributed environment is the synthesis of computational space, which is formed by instances of solvers. With a large number of nodes, launching applications manually can be quite time consuming. Therefore, it is necessary to provide the ability to automate the launch of the solvers by the supervisor. In this case, the means of remote task launching provided by a specific system are used. The created computational space can be used to solve the problem. In the solution process, it is necessary to distribute computations between solvers in order to maximize the efficiency of the application in a distributed environment. The exchange of data between solvers and the supervisor can be carried out by means provided for interaction with a specific node. If it is possible to establish a direct network connection, communication methods based on TCP / IP protocols are used, for example, the socket interface. In some cases, data exchange with applications is possible only through file transfer using the "middle" software.

Load balancing occurs at two levels:

- at the upper level, the supervisor distributes the computational load between solvers.
- at the lower (within one computational node), the solver distributes the work between methods intended for a particular type of computational node.

Both approaches of the search algorithm proposed above make it possible to automatically select an effective high-speed optimization method from the available list for solving a specific problem due to self-learning of the methods used.

The **appendix** lists the main modules of the developed software along with figures and tables.

CONCLUSIONS

In the dissertation work, numerical methods, algorithms, and software for solving several classes of parametric identification problems and problems of synthesis of control parameters for systems with both lumped and distributed parameters have been developed and studied. To solve these problems, a unified approach was used, which consists in introducing into consideration the concept of "zonal parameters" for both identification and control actions.

The main results of the dissertation work are as follows:

- A numerical method for solving problems of parametric identification of dynamic, in the general case, nonlinear objects with lumped parameters, described by systems of nonlinear differential equations with ordinary derivatives, is proposed and justified.
- An approach is proposed to the numerical solution of a class of inverse problems on determining the switching surfaces depending on the state of the dynamic process described by a discontinuous system of ordinary differential equations.
- An approach is proposed to numerically solve the problem of optimal control synthesis for objects described by systems of nonlinear ordinary differential equations for various types of input/output feedback and for various classes of zonal control functions with inaccurately given information on the values of the initial conditions and object's parameters.
- For decision-making systems on operating dynamic processes, an approach is proposed that combines the stages of parametric identification of a mathematical model and optimization of operating modes.

- The problems of identification of nonlinear coefficients of mathematical models and control synthesis with respect to specific objects with distributed parameters are solved using the approaches proposed in the work.
- A numerical method is proposed for solving the problem of identification of the hydraulic resistance coefficient, which depends on the modes of movement of hydrocarbon raw materials in a linear section of a pipeline, described by a system of hyperbolic equations.
- For objects with distributed parameters, on the example of controlling the heating process by a heat exchanger with a time delay in boundary conditions, based on continuous monitoring of the phase state of the object at certain points, an approach to the synthesis of control actions is proposed.
- On the example of problems of control of heating processes of a rod and a plate, an approach to the synthesis of control of lumped sources for objects with distributed parameters based on continuous monitoring of the phase state at certain points of the object is proposed.
- Mathematical software and programming packages for solving complex optimization problems based on interactive and automatic control of the unconstrained optimization software package using parallel computing on modern computer systems with multiprocessor/multi-core architecture has been created. The developed systems are equipped with an extensive library of optimization algorithms.

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The main results of the dissertation are published in the following papers:

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